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EDITORIAL

Permeability of Dust Cores

IN this number we publish an interesting letter from P. R. Bardell in which this question is discussed on the basis of experimental results obtained in the Research Laboratories of the General Electric Company. In the November editorial we referred to results obtained in America by Legg and Given which indicated that the permeability of dust cores was considerably higher than the value calculated on the assumption that the particles were uniformly distributed spheres or cubes. We expressed dissatisfaction with the explanation given and suggested that the results could only be obtained by giving up the assumption of uniformly distributed spheres or cubes and assuming some linking up of the particles in the form of threads in the direction of the field.

We are pleased to see that Mr. Bardell comes to practically the same conclusion. He says that the results agree with the calculated values if it is assumed that the particles are rectangular slabs of dimensions $10 \times 2 \times 1$ aligned in the direction of the field. It is interesting, but surprising, to know that photomicrographs show that such a flattening and alignment may be caused by the high pressures employed in the manufacture. We must confess that we find it rather difficult to picture the iron particle being changed into little strips five or ten times as long as they are wide, and obligingly arranging themselves in the desired direction, however great the applied pressure, but we write in ignorance of the manufacturing processes. It is as easy to picture a small extension in the desired direction squeezing out the insulating material in the end-to-end gaps and thus giving metallic continuity along the field.

It is fortunate that this unexpected practical departure from the theoretical values based on cubes or spheres is such as to rectify to a considerable extent the main disadvantage of dust cores, viz. their low effective permeability.

This alignment of the particles calls to mind the material known as Ferrocart which some of our readers may remember. It was developed by Hans Vogt and described by A. Schneider in the *Wireless Engineer* of April 1933. In this material the iron particles, which were stated to be ellipsoidal, were sprinkled on a strip of paper moving horizontally while their insulation was still liquid; the paper strip then passed through a solenoid carrying a direct current which aligned the particles along the strip before the insulating material dried and fixed them in the aligned position. A number of these paper strips were then built up with a hot liquid binder. The space factor was small and it was stated that the permeability could not be increased much above 18 without increasing the losses to an extent far outweighing the advantages of the higher permeability. The permeability now

obtained in dust cores is a multiple of this, and we look forward to the publication of further particulars of the permeability and losses of dust cores as now constructed and more detailed information as to the shape and alignment of the particles, on which the increased permeability depends.

With regard to the Ferrocart cores we drew attention in September 1941 to the fact that a sample core, received from the manufacturer in 1933, and kept in a cardboard box in a dry laboratory cupboard, had completely disintegrated into a loose heap of paper and what appeared to be rust, and that in less than eight years. We asked at the time if any readers who had inductances fitted with these cores would make measurements and let us know the results, but as we received no such information we presumed that Ferrocart cores had not been used to any extent in this country.

Wherever measurements of effective permeability of dust cores are given, measurements of the losses should also be given, in order to ensure that the increased permeability is not being obtained by sacrificing the very advantages because of which dust cores are employed.

G.W.O.H.

The Use of Standard Terms and Symbols

IN the October number of the *Proceedings of the Institution of Radio Engineers, Australia*, we are glad to see an article by a member, advocating a more rigid adherence to standardized terms, abbreviations and symbols. After discussing the resistor with its resistance, the conductor with its conductance, the reactor with its reactance, the inductor with its inductance, and the capacitor with its capacitance, the author says: "Other bad practices are concerned with abbreviations. For microfarad we may get mf, mfd, mF, μf, μF, and a similar range for micromicrofarad, μ is the accepted symbol for micro and F is the accepted abbreviation for farad. Therefore use μF and $\mu\mu$F. In typewritten matter use uF *and* uuF; these are to be preferred to mF and mmF. The 'diehards' will resist any encroachment on their bad habits:—'Why argue—you know what we mean, etc.' Even so, what is the objection to being correct? In this case, do we know what is meant? The abbreviation mF means millifarad, and mmf means magnetomotive force. The possibilities for confusion are real in this instance. None of those who use mF for microfarad would dream of using mV for microvolt."

Our Australian friend might have quoted a more horrible example for it is quite common in American journals and text books to find m used simultaneously for both milli and mega, and to see a frequency of 50 Mc/s referred to as a frequency of 50 mc.

There is one case in which one cannot but sympathize with the transgressor, and that is in the use of K instead of the standard k for a thousand. It would have been reasonable to have adopted small letters, μ and m, for submultiples of unity and capitals, K and M for the multiples, but the standardizing authorities decided on k and M, with the result that those who are careless in such matters are quite likely to use K and m, for a thousand and a million respectively.

G.W.O.H.

Correction.

In last month's Editorial the solidus was unfortunately omitted in the last expression in column 1, page 3. The last sentence here should have read "For the energy per cm^3 we have in one case $\mu H^2/8\pi$ and in the other $\kappa \mathfrak{E}^2/8\pi$."

H.F. RESISTANCE AND SELF-CAPACITANCE OF SINGLE-LAYER SOLENOIDS*

By R. G. Medhurst, B.Sc.

(Communication from the Staff of the Research Laboratories of The General Electric Company, Limited, Wembley, England)

SUMMARY.—This paper contains the results of high-frequency resistance and self-capacitance measurements on about 40 coils, wound with copper wire on grooved Distrene formers. The measuring instrument was a twin-T impedance bridge.

For coils whose turns are widely spaced, the high-frequency resistance measurements are in good agreement with the theoretical values of S. Butterworth.[3] For closely-spaced coils, the measured values are very considerably below those of Butterworth.

A table of values of high-frequency resistance of coils having various values of length/diameter and spacing ratio is derived from these measurements.

It is shown that a good approximation to the high-frequency Q of coils of the type measured is given by the simple expression

$$Q = 0.15\, R\psi\sqrt{f}$$

where R is the mean radius (cm), f the frequency (c/s) and ψ depends on the length/diameter and the spacing ratios. A table of ψ has been calculated.

Measurements of self-capacitance were made with one end of the coils earthed. These measurements show a very considerable divergence from the formula of A. J. Palermo[14] though they are in quite good agreement with other previous experimental work. The self-capacitance of coils of this type is shown to be substantially independent of the spacing of the turns. It is given by an expression of the form

$$C_0 = HD \text{ picofarads}$$

where D (cm) is the mean coil diameter and H depends on the length/diameter.

A table of H is given, based on these measurements.

1. Introduction.

A GREAT deal of theoretical and experimental work has been published concerning the resistances of coils and their variation with frequency. The experimental work, in general, suffers both from its restricted application and from uncertainty as to the absolute error inherent in the method of measurement. The theoretical work, even where it is in reasonable agreement with experiment, tends to produce complicated formulae which lead to very considerable computation. Even now, after over a quarter of a century of work on every type of coil that has been used or proposed, the present writer knows of no reliable data from which Qs of even the simplest coils can be easily and quickly predicted.

The only comprehensive theory extant is that of S. Butterworth[1–5]. He gave formulae which purported to cover single-layer solenoids, wound with round wire, for any frequency, coil dimensions and spacing of turns. The only restriction was that the number of turns had to be large, the case of few turns only being dealt with when the length of the coil was small compared with its diameter, and the turns were not too closely spaced. He suggested modifications to include multi-layer coils and coils wound with stranded wire. All these formulae only dealt with copper losses, dielectric losses being assumed negligible. Dielectric loss must, if necessary, be allowed for separately.

This theoretical work has become so generally accepted that it is quoted as a basis for calculation in standard reference books (see, *e.g.*, ref. 12, pp. 78–80). It will be shown experimentally that Butterworth's theory is only applicable to coils having widely spaced turns (roughly $d/s < 0.5$). More closely spaced coils have a lower high-frequency resistance than that predicted by Butterworth, the discrepancy increasing as the coil length decreases relative to the diameter. For coils having $l/D = 1$, when $d/s = 0.6$ Butterworth's value is too high by 15 per cent, when $d/s = 0.7$ by 25 per cent, when $d/s = 0.8$ by 55 per cent and when $d/s = 0.9$ by 190 per cent.

During the determination of high-frequency resistance and inductance it was necessary to make allowance for the self-capacitance of the coils measured. It was

* MS accepted by the Editor, July, 1946

thought that self-capacitances calculated from the formula of A. J. Palermo[13] would be sufficiently accurate for this purpose. However, it was found that use of Palermo's formula led to variations, larger than the experimental error, of the apparent measured inductances over quite a small frequency range.

Measurements of self-capacitance of a wide range of single-layer coils were consequently carried out. The results failed to confirm Palermo's claim that the self-capacitance varies steeply with the spacing of turns. They are, instead, in quite good agreement with previous work, which had shown the self-capacitance to be very nearly independent of d/s. For coils having length/diameter = 1, Palermo's formula gives results which are greater than the measured value by 4 : 1 when $d/s = 0.9$, and smaller than the measured value by 1.6 : 1 when d/s = 0.1.

2. List of Symbols

D	represents mean diameter of coil	(cm)
R	represents mean radius of coil	(cm)
l	represents overall length of coil	(cm)
n	represents number of turns	
d	represents diameter of each wire	(cm)
s	represents distance between centres of adjacent turns	(cm)
d/s	represents spacing ratio of turns	
ρ	represents resistivity of wire	(ohm-cm)
f	represents frequency	(c/s)
τ	represents power factor of material of coil former	
C_0	represents self-capacitance of coil	(pF)
L_s	represents equivalent series inductance of coil	(μH)
R_s	represents equivalent series resistance of coil	(ohms)
f_0	represents self-resonant frequency of coil	(c/s)
ϕ	represents ratio of h.f. coil resistance to resistance at same frequency of same length of straight wire.	
ψ	is a function of l/D and d/s, occurring in the formula for Q,	

$$i.e.,\ Q = 0.15\, R\, \psi \sqrt{f}$$

Where $Q = 2\pi f L_s/R_s$.

$$z = \pi d \sqrt{\frac{2f}{10^9 \rho}} = \frac{1}{\sqrt{2}} \frac{\text{wire diameter}}{\text{current penetration depth}}$$

3.—Butterworth's Work on Single-Layer Solenoids

3.1. S. Butterworth's series of papers on solenoidal coils are based on two sets of formulae which he developed for single-layer solenoids. In both cases there is no restriction on the frequency. The first[1] apply to coils having any specified number of turns, the turns being "not too closely spaced," and the coils having lengths small compared with their diameters. The second[3] were evolved as a consequence of some measurements by C. N. Hickman, which were made on coils outside the range of conditions assumed in the formulae of Ref. 1, and consequently failed to agree with the results predicted by these formulae[2,7]. This second group of Butterworth formulae extended the theory to coils having arbitary spacing ratio, and arbitary ratio of length to diameter. The number of turns, however, had now to be assumed to be large. Butterworth's method of deriving his second group of formulae is as follows:

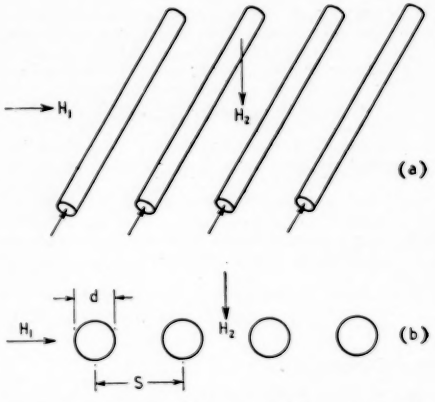


Fig. 1. *Views from above (a) and transverse to the wires (b) of Butterworth's infinite plane system of parallel wires.*

He took as his starting-point a system consisting of an infinite number of parallel wires of equal diameters, equally spaced and lying in the same plane as shown in Fig. 1, which shows a portion of the system seen from above and end-on. He set out to solve three problems, namely, to determine the losses in the system (*a*), when the wires carry equal alternating currents, i, flowing in the same direction; (*b*), when they are situated in a uniform alternating magnetic field H_1 parallel to the plane of the wires and perpendicular to their axes; and (*c*), when they are situated in a similar uniform field H_2 perpendicular to the plane of the wires. We shall call H_1 and H_2 the axial and transverse fields respectively.

Each of these problems involved the solution, by a method of successive approximations, of a set of an infinite number of linear equations, each containing an infinite number of variables. Butterworth's solutions are

contained in three tables, for a range of d/s from 0.1 to 1.0, in steps of 0.1 (*i.e.*, from widely separated turns to turns touching).

In order to apply these solutions to solenoidal coils, Butterworth worked out the field associated with the coil by adding to the field associated with an infinitely long solenoid a modifying field produced by its ends, considered as a circular disc of poles. This field was resolved into two components, one parallel to the axis of the coil (the axial field) and the other perpendicular to the axis (the transverse field), and the mean-square value of each over the length of the coil deduced.

Now, he pointed out that each short section of wire may, if the wire diameter is small compared with the coil diameter, be treated as part of a plane system, of the kind already considered. He considered that, as a sufficiently close approximation, the axial and transverse fields associated with the coil could be replaced by their mean-square values, these being considered to act uniformly along the length of the equivalent plane system, now taken as being infinitely long. Under such conditions, he showed that the total losses in the system equivalent to the coil could be obtained by summing the separate losses deduced from the solutions of his three problems.

His results for very high frequencies are summarized in a table which is reproduced here as Table I. The quantity tabulated is the ratio of the high-frequency resistance of a coil, assumed to have a large number of turns, to the resistance at the same frequency of a straight wire of the same length and diameter as the wire forming the coil. This straight-wire resistance can be calculated from a well-known formula. The variables of Table I are the ratio length/diameter of the coil, and the spacing ratio d/s.

" Very-high frequency " has to be defined in terms of the diameter and electrical constants of the wire. It is a frequency higher than that at which the current penetration depth is some arbitrary fraction, say, 1/10th, of the wire diameter. It is convenient to express the high-frequency resistance of a round wire in terms of a quantity z, which is defined by the relation

$$z = \pi d \sqrt{\frac{2f}{10^9 \rho}} = 0.107\, d \sqrt{f} \text{ for copper.}$$

Now, the current penetration depth in copper at frequency f

$$= \frac{1}{2\pi} \sqrt{\frac{10^9 \rho}{f}}$$

Hence, it follows immediately that the criterion, given above, that the frequency should be " very high " may be written in the form

$$z > 10/\sqrt{2}$$

i.e., $z > 7$ approximately.

3.2. Certain curious features are apparent in this Butterworth table. Particularly surprising are the high values that appear when the turns are closely spaced. Suppose for example, that we take a coil having a length/diameter $= 0.4$ and spacing ratio $= 0.8$—that is to say, with turns quite close. If we bring the turns a little closer to give a spacing ratio of 0.9, the length/diameter ratio being thereby only slightly changed, we are to expect the h.f. resistance to increase in the ratio of about 2.7 to 1. This would be surprising. For a spacing ratio of 1.0 the value infinity appears in the table; *i.e.*, when the turns are brought very close the h.f. resistance becomes infinitely large.

TABLE I

$\frac{d}{s}$	Coil Length/Coil Diameter											
	0	0.2	0.4	0.6	0.8	1.0	2	4	6	8	10	∞
1.0	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	3.41
0.9	18.2	17.5	16.1	14.6	13.2	11.9	8.02	5.27	4.39	3.96	3.78	3.11
0.8	6.49	6.32	5.96	5.57	5.23	4.89	3.91	3.20	3.04	2.97	2.92	2.82
0.7	3.59	3.53	3.43	3.29	3.17	3.07	2.74	2.61	2.51	2.51	2.50	2.52
0.6	2.36	2.35	2.32	2.29	2.26	2.23	2.16	2.15	2.14	2.16	2.16	2.22
0.5	1.73	1.74	1.75	1.75	1.75	1.76	1.77	1.85	1.85	1.86	1.86	1.93
0.4	1.38	1.39	1.41	1.42	1.44	1.45	1.49	1.56	1.57	1.59	1.60	1.65
0.3	1.16	1.19	1.21	1.22	1.22	1.24	1.28	1.34	1.34	1.35	1.36	1.39
0.2	1.07	1.08	1.08	1.10	1.10	1.10	1.13	1.16	1.16	1.17	1.17	1.19
0.1	1.02	1.02	1.03	1.03	1.03	1.03	1.04	1.04	1.04	1.04	1.04	1.05

Butterworth's theoretical values of the ratio of the h.f. coil resistance to the resistance at the same frequency of the same length of straight wire.

However, although this infinite value of the h.f. resistance holds for coils whose length varies from zero to a value indefinitely large, when the length actually becomes infinite there is a discontinuity, the h.f. resistance assuming a value 3.41 times the straight-wire resistance.

It will be useful to consider where, in Butterworth's calculation, these unexpectedly high values of h.f. resistance arise.

We have said that Butterworth evaluates separately three sets of losses, which he subsequently combines. These are (1) the losses due to the current in the wire under consideration and the currents in adjacent wires, (2) the losses due to the axial field (H_1 in Fig. 1) and (3) the losses due to the transverse field (H_2). Losses (1) and (2) show no surprising behaviour when the wires are closely spaced ; the abrupt rise in h.f. resistance for close spacing is all due to loss (3). Butterworth tabulates a quantity g, varying with d/s, which when multiplied by $(d/s)^2$ and a factor depending on the length/diameter ratio of the coil, gives the contribution of the transverse field losses to the total losses. This is reproduced as Table II.

Table II gives values, worked out by Butterworth, of the ratio of the losses in one wire of his infinite parallel-wire system, acted on by a transverse field, to those in an isolated wire acted on by the same field. The table shows the variation of this quantity with the spacing ratio. If we work through the theory again, taking account only of the two adjacent wires (besides the wire in question), we find for a spacing ratio of 1.0 that g comes out as 3.40 instead of ∞. Values of g for wider spacings will now lie between 3.40 and 1. Consequently, it is the more remote wires that, for close spacings, make the largest contribution to Butterworth's values of g given in Table II. In the case of a coil, the transverse field will be concentrated near the ends of the coil, that is to say, it will be effective over the last few turns. Consequently, the contribution of the effect of the transverse field to the total coil losses would be expected to be of the order of the value of g that we have just worked out, rather than of that given by Butterworth.

So far, we have only considered the results given by Butterworth for " very high frequency." We shall see later (Section 14) that it is also necessary to proceed with caution when trying to make use of his low-frequency results.

TABLE II

d/s	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
g	1.017	1.069	1.166	1.326	1.585	2.03	2.87	4.83	12.5	∞

That part of the resistance due to the transverse field rises steeply for spacing ratios over 0.7, becoming infinite for $d/s = 1.0$. In this latter case, the parallel-wire system becomes a continuous infinite metal screen at right angles to the field.

We can show, qualitatively, that Butterworth's high h.f. resistance values, for coils having closely-spaced turns, may be expected to be absent in practice. Butterworth has substituted for the actual transverse field of a coil a uniform field whose square is the mean-square value—obtained by a method of approximate integration—of the transverse field. This uniform field is supposed to be imposed on a system consisting of an infinite number of parallel wires, and we have to consider the effect on any one wire of this field as modified by the eddy-currents set up in that wire and in all other wires of the system. Now, we shall show that the high losses predicted by Butterworth in each wire of this system are due chiefly to the presence of the more remote wires.

4. Previous Experimental Work on the A.C. Resistance of Single-layer Solenoids

Butterworth, in his second paper[3], compares his theoretical values of the a.c. resistances of single-layer solenoids with two sets of experimental results, those of C. N. Hickman[7] and G. W. O. Howe.[6]

C. N. Hickman used coils wound with very thick wire (0.518 cm diameter), each coil being 96 cm in length, the ratio of length/diameter varying from 3 to 17. He used a d/s value of 0.86. He employed a bridge method of measurement, at frequencies of 1, 2 and 3 kc/s. The coils were wound on " well-seasoned wooden cylinders." Comparison of Butterworth's predicted values of resistance and Hickman's measured values shows that the former are from 2 to 13 per cent higher than the experimental.

G. W. O. Howe, using a thermal method, measured coil resistances at radio frequencies. He gave results for two coils, each having a length/diameter ratio of about 10, wound with wire of diameters 0.163 cm and 0.264 cm respectively. The respective values of d/s were 0.49 and 0.90. The results for the widely-spaced coil are close to those predicted by Butterworth. Those for the closely-spaced coil are lower than the theoretical values by $5\frac{1}{2}$ per cent at the low-frequency end of Howe's frequency range, and 20 per cent at the high-frequency end. Howe made his measurements on long coils because, at that time, no satisfactory formulae had been suggested for short coils. He pointed out that short coils, such as occur almost invariably in practice, would be expected to give results considerably different from those predicted by the long-coil theory, and he suggested a tentative modification of this theory to make it applicable to short coils.

Not a great deal of additional experimental work has been published, since the publication of Butterworth's papers. The most important is that of Dr. Willis Jackson.[8] He pointed out that Butterworth's formulae had not hitherto been satisfactorily verified. The two principal difficulties that he mentioned were that " precision uses of bridge networks are not available at radio frequencies," and that the losses in a tuning capacitor, which can at radio frequencies be comparable with those of the coil being measured, could not usually be measured separately. He avoided these difficulties by using an ingenious method suggested by E. B. Moullin, involving measurements on a number of coils having the same dimensions but wound with wire of different metals. The method is elaborate, and it would not be practicable to use it for measuring more than a small number of coils. It constitutes a check on an existing theory, rather than a method of making absolute measurements, and in addition Jackson found that it did not give results sufficiently accurate to allow more than tentative conclusions to be drawn. He worked at about 1 Mc/s, his coils having a spacing ratio of 0.63.

Sets of Q-meter readings, corresponding to coils of various dimensions, have been published from time to time. Typical of these are the values quoted in an article by Art H. Meyerson.[9] Results are given from 25 to 60 Mc/s in the form of a table, and some very irregular curves are based on the measurements. These results seem characteristic of the Q-meter rather than of the coils.

Of the methods of measurement so far considered, only the thermal method of Howe seems likely to give reliable results. The objection to Howe's method is the practical one of the length of time required for each measurement.

5. Description of Coils used in the Present Series of Measurements

The intention of the present series of measurements was to provide an empirical substitute for Butterworth's theoretical h.f. resistance table (Table I). We shall see that the conditions that need to be fulfilled before Butterworth's table becomes applicable—i.e., that z should be high and the number of turns large (see below)—tend to become incompatible over part of the range of the table, more especially in the bottom left-hand region. That is to say, practically useful coils cannot be wound such that they work at a large z value, have a large number of turns, have a wide turn spacing and have a small length/diameter. To cover this region, some compromise had to be made, and the results are of theoretical rather than practical interest. Butterworth, constructing his table exclusively from theoretical formulae, did not encounter this difficulty. However, most high-frequency single-layer coils actually used do fall within the region covered by the body of the table, which is the practically important region.

It is specified in Butterworth's table that the number of turns shall be large, though there is no indication of the effect on the h.f. resistance of employing a finite number of turns. We can form a very rough estimate of the order of magnitude of this effect as follows. If we assume that the part of the losses in each wire due to proximity effect is to be attributed largely to the two immediately adjoining wires, one on each side, the effect of using a finite number of wires will be, principally, to diminish the proximity effect in two turns, those at each end of the coil. As a first approximation, the proximity effect in each of these end turns will be diminished by half, since there is only one wire adjacent to each. Thus the total proximity effect losses will be diminished

by $2 \times \frac{1}{2} \times \frac{100}{n}$ %; i.e., by $\frac{100}{n}$ %.

Since, in practice, the proximity effect on each turn of more remote turns is not negligible, this expression will be too low. Since, however, proximity losses form only part of the total losses, this expression should give the order of magnitude of the effect of using a finite number of turns. It is not in violent disagreement with Butterworth's figures for very short coils, of not too close spacing, having a finite number of turns.

From this consideration, it was decided that the minimum number of turns that would be used was 30. In practice, the number of turns ranged from 30 for the smallest length/diameter coils to 50 for the largest.

The condition that the number of turns shall be large may, over part of Butterworth's table, be difficult to reconcile with the condition that the frequency shall be high, or, more correctly, that z should be large. From the expression for z, i.e.,

$$z = 0.107\, d\sqrt{f}$$

it is evident that both the wire diameter and the frequency must be as large as possible. As regards the frequency, since as we shall see later, we shall want to make use of a large tuning capacitance, in order to work at a high enough frequency our coil inductance must be as low as possible. That is to say, we must use the smallest permissible coil diameter and the smallest number of turns. The minimum number of turns has already been decided. As for the coil diameter, since the number of turns has a lower limit, and the spacing of turns and the ratio of coil length to coil diameter are determined by the position of the coil in Table I, we can only decrease the coil diameter by decreasing the diameter of wire. But it is evident from the form of the expression for z, that, for satisfying the " high-frequency " criterion, a large d is more important than a large f.

Specifically, since for a given number of turns and a given spacing ratio, the length of coil, l, is proportional to the diameter of wire, d, then the coil diameter D, for a given l/D ratio, will also be proportional to d. But f is proportional to $1/\sqrt{L}$, i.e., to $1/\sqrt{D}$, and hence to $1/\sqrt{d}$. So $\sqrt{f}$ is proportional to $1/(d)^{\frac{1}{4}}$, and hence z is proportional to $(d)^{\frac{3}{4}}$.

It is apparent that the upper limit to d is determined by the maximum permissible coil diameter. In the present work, the largest formers used had a diameter of 2½in. This involved the use of wires of 18 and 20 S.W.G. for most of the coils. Furthermore, since for a given maximum coil diameter, the upper limit of z becomes increasingly severely diminished with decreasing value of length/diameter, no coils were constructed having a length/diameter less than 0.4.

The coils were wound on Distrene rod, diameters ranging from 2½in to ⅝in. In one case, to attain a sufficiently high length/diameter ratio with a spacing ratio of 0.9, a coil of 20 S.W.G. double-silk-covered wire was wound on a ¼-in diameter former. The special virtues of Distrene rod for the present purpose are its low power factor (about 0.0003) and, from the mechanical point of view, the ease with which it can be grooved and the ends of the coil anchored.

The effect of dielectric losses on the total losses may be estimated if we assume that the coil turns are completely embedded in the former. Since in reality, each turn is roughly half surrounded by air and half by former material, we may expect that this assumption will cause us to over-estimate the actual dielectric losses. With this limitation, the percentage increase in the copper-loss series resistance due to dielectric losses is given by

$$\frac{\tau\omega^3 L^2 C_0}{R} \times 100\ \%$$

where τ is the power factor of the former material,
ω is the angular frequency,
L is the coil inductance,
C_o is the coil self-capacitance,
R is the equivalent series resistance due to copper losses.

This may be written as

$$\tau Q(f/f_o)^2 \times 100,$$

where f is the working frequency and f_o the self-resonant frequency,
or as

$$\tau Q(C_o/C) \times 100$$

where C is the tuning capacitance at the working frequency. Q, for the coils measured, was never higher than about 250, and C_o was usually about 3 or 4 pF. Minimum C was about 850 pF. Thus, the maximum value of the percentage dielectric losses that we may expect is

$$\frac{0.0003 \times 250 \times 4 \times 100}{850}$$
$$= 0.035\ \%\ \text{approx.}$$

The ends of the wires (mostly 18 and 20 gauge copper wires) were soldered to leads

of 12 or 14 gauge copper wire, which had been previously tinned and sunk about $\frac{1}{4}$in into the former (using the heat of a soldering iron pressed against the lead just above the former). There is a tendency for the leads to twist in their holes, if they are not bent carefully.

For spacing ratios up to 0.8, the formers were grooved and the coils were wound with bare wire. For a spacing ratio of about 0.9, double-silk-covered wire was wound on ungrooved formers, the turns being as close as possible. This procedure gave spacing ratios ranging from 0.885 to 0.92. One coil (length/diameter = 1.30) was wound with single-silk covered 20 gauge wire (d.s.c. wire, from which the outer layer was stripped) on an ungrooved former, a spacing ratio of about 0.95 being obtained.

There is an appreciable tolerance on wire sold by gauge number, and in any case, the wires are stretched before winding to remove kinks. Consequently, the wire diameter for each coil was taken as the mean of three or four roughly equally spaced measurements, by a centimetre micrometer, along the length of the wire, these measurements being made during the winding.

Each coil was dried for about twelve hours in a desiccator, before measurement.

6. Use of the Twin-T Impedance Measuring Bridge

The twin-T Impedance-Measuring Circuit manufactured by the General Radio Company, Cambridge, Mass., is one of the most recent developments in the technique of measuring high-frequency impedances. Detailed accounts of the theory and its practical application are given in references 10 and 11. We shall very briefly describe its use for the measurement of high-frequency coil impedances.

The twin-T operates over a frequency range from 460 kc/s to 30 Mc/s. It is a null instrument, a balance being obtained by the adjustment of two capacitors. One of these capacitors is calibrated in micro-mhos, and, after multiplying by a factor involving the square of the frequency, gives the effective parallel conductance of the measured impedance. The other, calibrated in picofarads, gives directly the effective parallel capacitance.

The capacitance dial has a range of from 100 to 1,100 pF, and it is calibrated at intervals of 0.2 pF. The scale of the conductance dial is not linear, the calibration intervals ranging from $2\frac{1}{2}$ to 10 per cent of the measured conductance. This dial can usually be read to 1 per cent or better.

In the present series of measurements, the oscillator consisted of a Marconi Signal Generator, covering a frequency range from 85 kc/s to 25 Mc/s. A number of receivers were used from time to time, the necessary qualifications being that the receiver should be well shielded, that it should have a sensitivity of the order of 1 to 10 microvolts, and that it should be provided with a local oscillator to beat with the incoming signal, so as to produce an audible note.

7. Method of Measurement

Each coil was measured over a small band of frequencies at the low-frequency end of the available working range; i.e., at frequencies at which it tuned with capacitances of 800–1,000 pF. There were two reasons for working at the lowest available frequencies: (1) because the conductance-dial reading falls off rapidly as the frequency increases (being inversely proportional to $f^{3.5}$, for the present type of coil) and the error in the reading becomes correspondingly larger, and (2) because the effect of the resistance of the twin-T tuning capacitor is minimum at the lowest frequencies.

Each set of measurements was entered up on a standard sheet. We have reproduced, as a typical example, that relating to coil No. 31. Nine measurements were carried out on each coil, the first at a frequency at which the coil tuned with about 1,000 pF, and the remainder at frequencies increasing progressively by the smallest dial intervals of the signal generator. It was thought preferable to perform each measurement at a different, rather than at the same, frequency to avoid, if possible, systematic errors. The values of $Q/\sqrt{f}$ thus obtained, after various corrections had been made (see over), usually showed a spread of from 3 to 5 per cent.

Various necessary formulae were reproduced, for convenience, on each sheet.

8. Corrections Applied to the Twin-T Measurements

Six corrections had to be applied to the original measurements. Four of them will be described in this section, and the remaining two will be given a section each.

TABLE III

29 . 8 . 45.

Coil No. 31. Mean temperature, 20.5° C.

Freq. Mc/s	C_{p1} (pF)	C_{p2} (pF)	Conductance dial reading	$C_{p2}-C_{p1}+3.4$ (pF)	Corrected conductance dial reading	$-\delta G$	True conductance (μmhos)	Q	$\frac{Q}{\sqrt{f}}$	L (μH)
0.78	100	1,080.3	34.9	983.7	21.2	0.1	21.1	228	0.258	42.32
0.79	100	1,056.2	32.9	959.6	20.5	0.1	20.4	233	0.262	42.29
0.80	100	1,031.8	32.0	935.2	20.5	0.1	20.4	230	0.257	42.32
0.81	100	1,007.9	30.8	911.3	20.2	0.1	20.1	231	0.257	42.36
0.82	100	984.8	29.1	888.2	19.6	0.1	19.5	235	0.260	42.41
0.83	100	962.6	28.0	866.0	19.3	0.1	19.2	235	0.258	42.46
0.84	100	943.5	27.0	846.9	19.1	0.1	19.0	235	0.256	42.39
0.85	100	923.4	26.0	826.8	18.8	0.1	18.7	236	0.256	42.40
0.86	100	904.0	25.2	807.4	18.6	0.1	18.5	236	0.255	42.42
								Mean	0.2577	42.37

No. of turns = 40

Wire gauge = 20 s.w.g.

D = 5.19—0.09 cm = 5.1 cm

R = 2.55 cm

length = 7.01 cm

$\frac{\text{Length}}{\text{diameter}}$ = 1.375

d = 0.0910 cm

$s = \frac{6.92}{39}$ cm

$\frac{d}{s} = \frac{0.0910 \times 39}{6.92}$ = 0.513

$Q = \frac{6.283\,fC}{G}$

$L = \frac{0.02533}{f^2 C}$

$R_s = \frac{6.283L}{Q/\sqrt{f}}\sqrt{f}$

$\phi = \frac{[(\text{resist})/10^{-6}\sqrt{f}] \times d}{0.5218\,Rn}$

$Rexp.$ = 1,033 × $10^{-6}\sqrt{f}$ ohms

Resistance of leads = 7 × $10^{-6}\sqrt{f}$ ohms

Resistance of coil = 1,026 × $10^{-6}\sqrt{f}$ ohms

ϕ = 1.745 = 1.743 at 20° C = 1.71 at 20° C $f \to \infty$

Resistance of leads = 0.1661 × $10^{-6}\frac{l}{d}\sqrt{f}$

where $l = \frac{1}{2}$ (total length of leads).

8.1. The conductance-dial reading has to be multiplied by a factor of the form $\left(\frac{f}{f_o}\right)^2$, where f is the working frequency and f_o is either 1, 3, 10 or 30 Mc/s according to the frequency range which is used. The results of this correction are in column 6 of the measurements sheet.

8.2 The corrected conductance value thus obtained has to be further corrected on account of the resistance, R_c, of the metal structure of the main capacitor. According to Sinclair's paper,[11] R_c, in a twin-T whose residual parameters were measured by him, had the value 0.025 ohm at 30 Mc/s. As will be shown later, even a large departure from this value in the machine actually used, will not seriously affect the present measurements. Assuming this resistance to be proportional to the square root of the frequency, the error it introduces into the conductance reading can be corrected by adding algebraically a factor δG, which has the form

$$\delta G = -\{0.184(f)^{2.5}\}(C_{p2}^2 - C_{p1}^2) \times 10^{-6}\ \mu\text{mhos}.$$

f represents the frequency (Mc/s), and C_{p1} and C_{p2} are respectively the initial and final settings of the tuning capacitor (pF).

This is a very cumbersome expression to use, if one has to work out a large number of values of δG. The labour of evaluation can be considerably decreased by using a graphical procedure. The first part of the expression, $0.184(f)^{2.5}$, is plotted on double-logarithmic graph paper (Fig. 2), for a range of f from 0.1 to 10. The second part, $(C_{p2}^2 - C_{p1}^2) \times 10^{-6}$, is plotted as the third line of a nomograph (Fig. 3) the two base lines being C_{p1} and C_{p2}, each ranging from 100 to 1,150 pF. δG is now obtained as the product of the two quantities read off from Figs. 2 and 3.

As an example: for coil No. 31, at a frequency of 0.78 Mc/s C_{p1} was 100 pF, and C_{p2} 1,080.3 pF.

Now, $0.184(f)^{2.5} = 0.184(0.78)^{2.5} = 0.1$ from Fig. 2

and $(C_{p2}^2 - C_{p1}^2) \times 10^{-6} = 1.16$ from Fig. 3

so $\delta G = -0.1 \times 1.16 = -0.1\ \mu$mho approx.

While the value of R_c, for the particular twin-T used, was probably not identical with that of the twin-T measured by Sinclair, it was not thought necessary to repeat Sinclair's measurements. There were two reasons for this: (1) the correction for R_c, at the frequencies used, is a small fraction of the

measured conductance (usually of the order of 1 or 2 per cent), and the correction would not be appreciably affected by any likely deviation of R_e from Sinclair's value, and (2) the value assumed for R_e, at frequencies of the order of 1 Mc/s is, in any case, an approximation, since R_e has to be measured at a frequency round about 30 Mc/s and the doubtful assumption is made that R_e will vary precisely as the square root of the frequency.

8.3 The correction for losses in the leads is most conveniently applied by subtracting it from the equivalent series resistance. The expression for the h.f. resistance of a straight

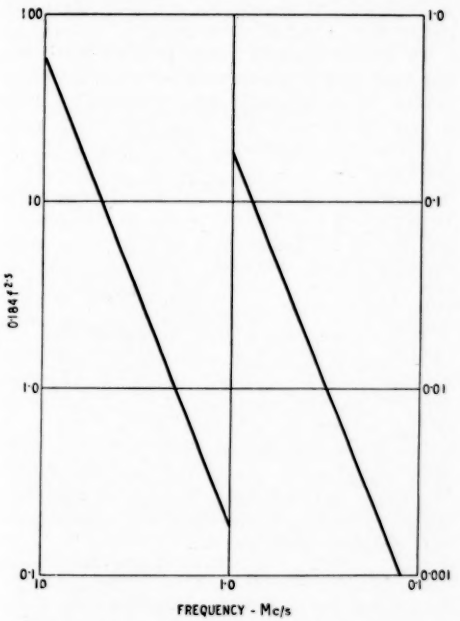


Fig. 2. *Correction for R_e (1).*

isolated copper wire was considered a good enough approximation to this loss. The correction was usually less than 1 per cent of the total series losses.

8.4 Temperature variation of the dimensions of the coils will not produce an appreciable effect on the series resistance over the temperature range (from about 16° to 26° C) encountered. However, temperature variation of resistivity has to be taken into account. Taking the temperature coefficient of resistivity of copper as 0.0039 at 20° C, we have to multiply the apparent value of ϕ (the ratio of the coil resistance to the resistance of the same length of straight wire at

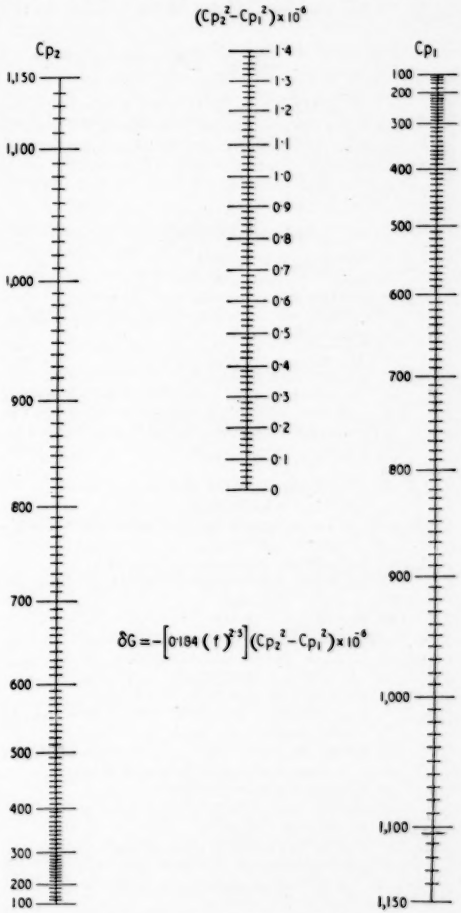


Fig. 3. *Correction for R_e (2).*

the same frequency) by one of the following factors.

$$\frac{1}{\sqrt{1 + 0.0039(t - 20)}} \quad \text{if } t > 20° \text{ C}$$

or $\sqrt{1 + 0.0039(20 - t)}$ if $t < 20°$ C

where t°C is the temperature of measurement.

8.5 The remaining two corrections are (5), for coil self-capacitance and (6), a frequency correction to the final value of ϕ. We shall consider each of these in a separate section.

(*To be concluded*)

For references see end of article in March issue.

RECTANGULAR WAVEGUIDE SYSTEMS*

Bends, Twists, Corners and Junctions

By N. Elson, M.A., Grad.I.E.E.

SUMMARY.—The elements most commonly required in the construction of a waveguide transmission system are discussed, with particular reference to the methods which have been adopted to ensure complete transmission of the power through each element. The paper includes the main aspects of the theory, design and methods of construction of bends, twists, corners and junctions for use in rectangular waveguide systems.

LIST OF SYMBOLS.

a = width of waveguide.
b = height of waveguide.
E = electric field vector.
f = specific admittance = admittance $/(1/Z_0)$.
H = magnetic field vector.
j = $\sqrt{-1}$
l = length along the axis of the tube.
J_p = Bessel function of the first kind of order p. The addition of a prime denotes differentiation with respect to the argument.
k = π/a.
N_p = Bessel function of the second kind.
p = Angular wavelength constant.
$r.\exp.(j\phi)$ = complex voltage reflection coefficient.
r, ϕ, z = cylindrical polar co-ordinates.
S = standing-wave ratio = minimum/maximum voltages.
Z = impedance.
Z_0 = characteristic impedance of line.
z = Z/Z_0 = specific impedance.
α = attenuation constant.
β = wavelength constant = $2\pi/\lambda_g$.
β_0 = wavelength constant for free space propagation = $2\pi/\lambda$.
ϵ = permittivity of the medium ($= 10^{-9}/36\pi$ farads/metre for free space).
λ = free-space wavelength.
λ_c = critical wavelength.
λ_g = guide wavelength.
λ_s = guide wavelength in a straight guide.
μ = permeability of the medium ($= 4\pi \times 10^{-7}$ henrys/metre for free space).
ω = angular frequency.

1. Introduction

THE use of centimetric waves in radar equipment has led to the replacement of the conventional coaxial or twin-line transmission line by the hollow-tube waveguide as a means of conveying the radio energy from its source to its desired destination. In most practical applications the transmission path cannot be straight, but will be constrained by other mechanical requirements as in any other plumbing† system. These requirements have opened a wide field of waveguide research, and this paper is concerned with some of its results.

It has previously been pointed out[1] that the best type of waveguide for most purposes is one of rectangular cross-section, of such a size that only the $H_{1,0}$ mode can propagate. The elements here described are therefore only of this type of cross-section. The restriction of the modes of propagation to only the one lowest mode is ensured by the use of a cross-section having an internal breadth lying between one-half and one free-space wavelength, and a height of less than one-half of a wavelength.

It is unnecessary here to recall the full field theory of such a waveguide[2], but some reference to evanescent modes is required for an adequate understanding of the effect of discontinuities in a waveguide. For each mode in a waveguide there is a characteristic field distribution across the cross-section of the tube, and this field propagates down the guide, according to the usual wave expression (in complex notation) :—

$$\exp.(j\omega t \pm j\beta l) \quad .. \quad .. \quad .. \quad (1)$$

where l is the distance along the axis of the tube and β is the wavelength constant. β is the imaginary part of the propagation constant.

Also

$$\beta = 2\pi/\lambda_g \quad .. \quad .. \quad .. \quad .. \quad (2)$$

where the guide wavelength λ_g is given by

$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda^2} - \frac{1}{\lambda_c^2} \quad .. \quad .. \quad .. \quad (3)$$

λ being the wavelength for propagation in the unbounded medium; while λ_c is the critical wavelength dependent only on the tube size and the mode, being largest for the lowest mode, and steadily decreasing for higher modes.

* MS. accepted by the Editor, June 1946.

† The appropriately descriptive term "Waveguide plumbing," of American origin, is now commonly used in this country.

The tube has been chosen so that the free-space wavelength λ is greater than the critical wavelengths of all modes except the first. Thus, for all these higher modes the expression (3) becomes negative, with the result that the guide wavelength, and hence the wavelength constant, become pure imaginary quantities. Writing for these cases $\beta = -j\alpha$, where α is a real quantity, the wave expression (1) becomes

$$\text{exp. } (j\omega t \pm \alpha l) \quad .. \quad .. \quad .. \quad (4)$$

This means that any such higher modes, if excited at any point in the tube, will not be propagated as waves but will appear as fields of uniform phase with amplitudes decaying exponentially along the axis of the tube. Such " static " modes are termed evanescent modes. It may be noted that the higher the mode the higher is the rate of decay.

2. Mismatch in a Waveguide

A waveguide system, which is capable of transmitting energy in only one mode, may conveniently be considered in terms of the conventional transmission line theory. It has been shown, notably by Schelkunoff[3], that the transmission-line equations are applicable to a waveguide, if voltage and current are replaced by appropriate functions of the electric and magnetic fields respectively. In this way a waveguide may be said to have a characteristic impedance which, like that of a line having two conductors, is dependent on the shape and size of its cross-section, but which is unlike in being dependent also on the ratio of its dimensions to the free-space wavelength. Similarly, discontinuities in the waveguide, such as changes in shape or constitution from the straight uniform tube, may be regarded as impedances in series or in parallel with the transmission line ; or, if their effect is distributed, as sections of line of different characteristic impedance and propagation constant ; or as combinations of both. In determining the electrical length of a portion of waveguide, the propagation constant must be taken as that appropriate to the phase velocity of the guided plane waves, as given by equation (2).

It is, however, unnecessary initially to enquire closely into such definitions of impedance from the electromagnetic field conditions, but to regard impedances only as useful expressions which are formally related to the voltage reflection coefficient. At any point in the waveguide sufficiently far from a discontinuity for any evanescent modes there excited to have decayed to negligible proportions, there can only be two waves present. One is the forward travelling wave and the other the backward travelling, or reflected, wave. The field may therefore be represented by the field distribution characteristic of the propagating mode multiplied by the wave expression :—

$$\text{exp. } (j\omega t - j\beta l) + r.\ \text{exp. } (j\omega t + j\beta l + j\theta) \quad (5)$$

where $r.$ exp. $(j\theta)$ is the reflection coefficient at the point $l = 0$, which may be arbitrarily chosen, giving the amplitude and phase of the backward travelling wave relative to the forward travelling wave at that point. In transmission-line theory this voltage reflection coefficient is readily related to the total impedance Z at the point relative to the characteristic impedance Z_o of the line, by the expression

$$r.\ \text{exp. } (j\theta) = \frac{Z - Z_o}{Z + Z_o} = \frac{Z/Z_o - 1}{Z/Z_o + 1} = \frac{z - 1}{z + 1} \quad (6)$$

Thus in waveguide applications, the specific impedance z may be regarded as an alternative parameter expressing the reflection occurring at the point considered, and without need to refer it to any other means of definition. The quantity which is directly measured is the ratio of the minimum and maximum amplitudes of the electric field along the guide and this standing-wave ratio is expressible by the equation

$$S = \frac{1 - r}{1 + r} \quad .. \quad .. \quad .. \quad .. \quad (7)$$

(Some writers use the reciprocal of this quantity : others, mainly American, express this ratio in decibels, a convenient device if the measuring instrument is logarithmically calibrated.)

The problem of transmitting all the power through the system thus corresponds exactly to the better-known transmission-line matching problem. At a discontinuity, where the reflection coefficient is $r.$ exp. $(j\theta)$, the power reflected is r^2 relative to the incident power. Thus a standing wave ratio of 0.5 corresponds to $r = 1/3$, or an 11% loss of power by reflection.

In radar applications, however, two other factors are important in deciding the degree of matching that is necessary. Firstly, the presence of standing waves produces increased fields, which may cause breakdown

of the air dielectric and consequent sparking across the guide. Secondly, and more important, is that pulsed-magnetron transmitters require a well-matched load if the efficiency and the pulse shape are to be maintained at their optima. It has therefore been normal practice to keep the mismatch due to a discontinuity in a waveguide down to a corresponding standing-wave ratio of 0.95 or better.

3. Discontinuities in Waveguides

The effect of a discontinuity in a waveguide is, in general, to produce a reflected wave. The mechanism of this effect can perhaps best be understood by reference to a particular and simple case. Consider a thin metal iris, of the type shown in Fig. 1(a),

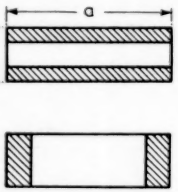


Fig. 1. *Capacitive and inductive irises are shown at (a) and (b) respectively.*

placed across the waveguide. Such an iris produces a reflected wave corresponding to that produced by a capacitor connected across the analogous transmission line [see Figs. 2(a) and (b)]. It is a device which has practical uses, and is commonly known as a capacitive iris or window. To consider its effect separately, assume that the terminating end is perfectly matched by an absorbing or radiating device.

On a conducting surface the component of the electric field parallel to the surface must vanish. These conditions must be satisfied over the surfaces of the iris in addition to the surfaces of the walls of the tube. Now the latter requirement will be satisfied by any of the modes corresponding to the tube size. By suitably combining, on each side of the iris, a series of these modes with appropriately related amplitudes and phases the former requirement can also be met. Of all these modes only the one mode will propagate as a wave to or from the iris, the others will be evanescent, and quite clearly will be exponentially *decreasing* in amplitude away from the iris on both sides. The propagating mode may be regarded as three waves, one travelling forwards on both sides of the iris, and two waves each travelling outwards from the iris. The series of modes will be an infinite series, although considerations of symmetry will limit the excited modes to those of the form $H_{1,n}$. The sums of the displacements of all these modes must be identical on both sides of the iris: the series of modes on each side are therefore as represented diagrammatically in Fig. 2(c).

If these amplitudes refer to the voltage (or its waveguide analogue) it is clear that the iris may be regarded as a shunt impedance across the line. Since for good conductors there can be no appreciable power loss by absorption in the metal, this impedance must be purely reactive: the complete analysis shows it to be capacitive. Such a capacitor across a transmission line would act as a store of energy, taking in energy during one-quarter of a cycle and giving it out again in the next quarter: in the waveguide it is the evanescent modes which act in exactly similar fashion as storers of energy, the transverse electric field being in phase quadrature with the transverse magnetic field. The one major difference between the iris in the waveguide and the capacitor across the line is the different degree to which the stored energy is localized. Unless recognized this can lead to incorrectly interpreted results when discontinuities occur closely together in a waveguide. If the evanescent modes due to the one discontinuity have not decayed to negligible proportions in the vicinity of the other, the fields must undergo

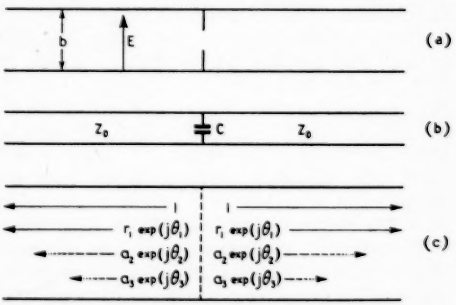


Fig. 2. *An iris across a waveguide (a) is equivalent to a capacitor across a transmission line (b). The component waves in the guide are illustrated at (c).*

a modification from those which would exist if the discontinuities were widely separated in the waveguide. In the transmission line equivalent this may be interpreted as the presence of a mutual impedance effect.

In a similar manner the window of the form shown in Fig. 1(b) turns out to be

representable as a shunt inductance. In both cases the reactance will depend on frequency in a more complicated manner than that of its circuit analogue. A discontinuity which is not confined to a very short length along the axis of the tube may similarly be regarded as distributed loading of the line by shunt or series impedances with appropriate mutual impedance effects. In this way a length of waveguide of different cross-sectional size interposed in the standard waveguide system may be regarded as a length of line of different characteristic impedance and propagation constant. However since the steps at the junctions between the guides of different size will be comparable with the wavelength, they must also be included in the equivalent circuit as shunt reactances at the junctions.

Any discontinuity may therefore be expressed by an equivalent transmission line circuit involving suitable combinations of lumped and distributed impedance ; their values being separately expressible as functions of frequency. It is, however, often more convenient, as in circuit theory, to consider such a section of line as a general four-terminal network or quadripole. The essential linearity between the magnetic and electric fields in each mode ensures the applicability of the usual bilinear relationships. The behaviour of a four-terminal network can be expressed in general by three complex parameters. Out of the region of evanescent modes there will be four waves present in the general case, one forward- and one backward-travelling wave on each side of the quadripole. The three complex parameters used to express the behaviour of a quadripole are thus directly related to the three complex coefficients which express the amplitudes and phases of three of these waves relative to the fourth.

4. Methods of Matching

In twin-conductor transmission lines, various methods of matching-out reflections have been adopted, amongst these being the placing of a shunt reactance of correct size at the appropriate place in the line. This method is applicable to waveguides, the reactance usually taking the form of an inductive iris of the type previously mentioned, capacitive irises not being favoured for this purpose owing to their greater tendency to spark over with high powers. This method of matching suffers from the disadvantage that it is sensitive to frequency, because the electrical length of the line between the discontinuity and the matching iris changes with frequency, and there are also changes in the reactance of the iris and the reflection coefficient at the discontinuity. It rarely happens that these changes are adequately compensatory to each other ; and, in general, matching over a wide frequency band can best be obtained by making modifications at and to the discontinuity itself. This may be regarded as a means of employing usefully the mutual impedance effects to obtain a greater bandwidth, in a similar manner to that in which they are used in the design of circuit band-pass filters.

This method of matching *in situ* has been applied to the design of reflectionless curved bends, twists, corners and junctions, which will now be considered in turn.

5. Curved Bends

An obvious way of turning a rectangular waveguide through an angle is by means of a curved bend. Bends in both principal planes may be required : those in which the long sides of the guide, and hence the magnetic field vector, are in the same plane are termed H-bends, while those with the electric field everywhere in the plane of the bend are termed E-bends. In both cases it may be shown that waveguide modes exist for uniformly curved bends of rectangular cross-section, having wave fronts in the radial planes and travelling round the curve with uniform angular velocity. These modes tend to the straight waveguide modes as the radius of the bend is made large (see Appendix). Thus, the curved bend may be regarded as a length of transmission line of different characteristic impedance and propagation constant, with the addition of shunt reactances at each end representing the reflections due to the junctions alone. The effect of the different characteristic impedance may be eliminated by making its length equal to an integral number of half curved-guide wavelengths. This requirement will determine the mean radius of curvature for a tube of given cross-section (see Appendix). The effect of the reflections at the junctions will be additive under these conditions, but they are generally small enough to be neglected. A partial correction may be obtained when necessary by making a small change in the mean radius, most satisfactorily determined experimentally.

Curved bends of small radius have not

been widely used in this country, owing to the development of the more compact angular bends or corners discussed later in this paper. Bends of large mean radius, greater than a free-space wavelength, give little reflection (standing-wave ratio better than 0.95) even at their worst electrical length. Where space permits they are quite

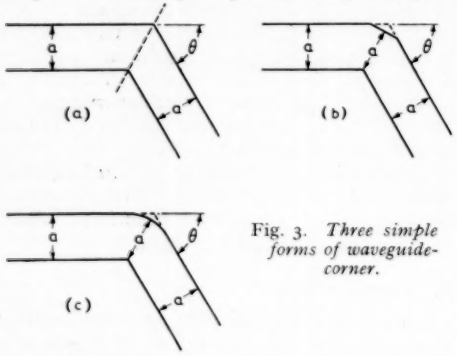


Fig. 3. *Three simple forms of waveguide-corner.*

convenient, and have been made on 3-cm wavelengths by bending copper rectangular waveguide tubing, the cross-section being preserved by filling with a low melting-point alloy before bending. Bends of small radius have some advantage over corners at short wavelengths since they will then be made of machined parts, and the necessary tolerances are more easily maintained than in the case of corners, which can only be made by fabrication or die-casting. If the curved bend is made in two machined parts which are to be bolted together it is advantageous to cut the curved guide into its two parts in the plane of minimum transverse current (see Appendix). If the two halves are to be soldered together the break may be made where convenient.

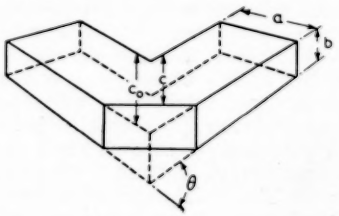
An E-bend of 135° for use on wavelengths of about 3 cm, whose mean radius is about one-half of an inch, has been developed for use in a British radar equipment, and produces a standing-wave ratio in an otherwise matched guide no worse than 0.95 over a frequency bandwidth of $\pm$ 3%.

6. Twists

It is often convenient to be able to twist a rectangular waveguide through an angle, usually a right-angle, about its axis, so as to orientate the plane of polarization. Such a twist has the same type of equivalent transmission line circuit as the curved bend: the change in characteristic impedance and the junction effects are, in general, much smaller and the overall reflection for practical cases is negligible. This assumes that the cross-section is maintained during the twist: indeed this of itself sets a practical limitation to the rate of twist which ensures that the reflection is negligible.

7. Corners

The term corner has been used by the author to describe bends which are essentially angular rather than smoothly curved. The first type of corner to be tried was the obvious sharp angled bend of Fig. 3(a). The reflection coefficients for such bends, for the two planes of bending, and different angles of bends, are shown in the graphs of Figs. 4 and 5 with the abscissae equal to unity. The modified bends shown in Figs. 3(b) and (c) were next tried, and some improvement was obtained. Another idea tried at the same time was that of using two corners of the simplest type, each of half the angle, placed



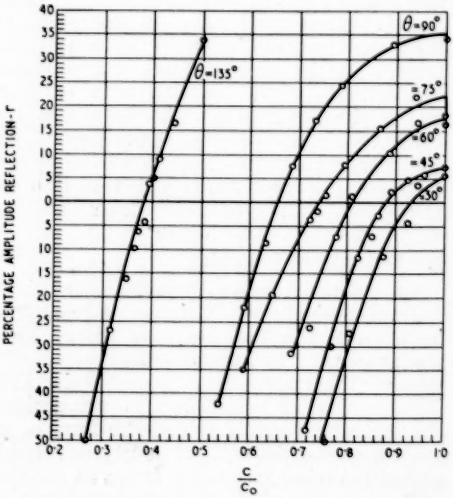
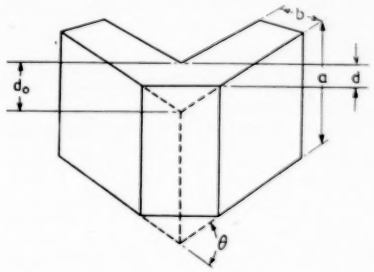


Fig. 4. *H-bend corner for a rectangular guide for $H_{1,0}$ waves. $\lambda = 10.84 \pm 0.02$ cm; $a = 7$ cm; $b = 3.25$ cm; $C_0 = a \sec \theta/2$; internal dimensions.*

an appropriate distance from each other to cause mutual cancellation of their separate reflections. It was demonstrated that this method could be satisfactorily used where space permitted, but since the spacings used in the early experiments were greater than



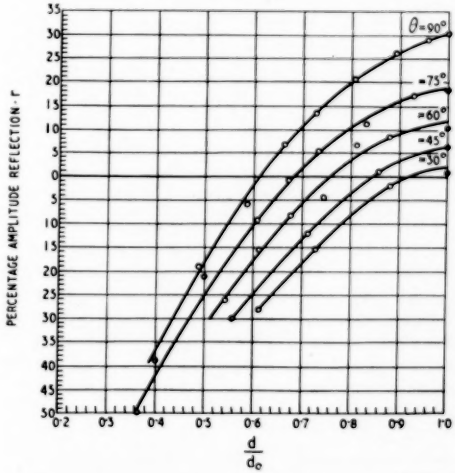


Fig. 5 (above). *E-bend corner for a rectangular guide for $H_{1,0}$ waves. $\lambda = 10.84 \pm 0.02$ cm; $a = 7$ cm; $b = 3.25$ cm; $d_0 = b \sec \theta/2$; internal dimensions.*

Fig. 6 (right). *Empirical relation between c and d of Figs. 4 and 5 and the angle of bend θ. For H-corners, $c = 7 - 0.49 \sin \theta$ and for E-corners, $d = 3.25 - 0.43 \sin \theta$.*

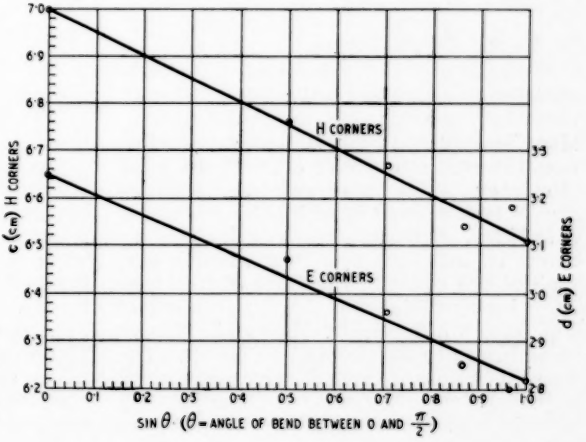
their minimum value the conditions were not at their optimum for maximum bandwidth.

At this stage attention was devoted to the chopped-off corner of Figs. 4 and 5 where the dimension across the corner was to be varied experimentally in the hope of obtaining a reflectionless corner. Figs. 4 and 5 show the effect of varying the dimension across the corner for bends in both planes and of various angles. The earlier polygonal corner of Fig. 3(b) corresponds to a value of the abscissa equal to $\cos \theta/2$. Fig. 6 shows an attempt to relate empirically the optimum width across the corner to the angle of bend for the particular cross-section and wavelength used. For a waveguide whose cross-section is $a \times b$, the relations are adequately represented for angles of bend up to a right-angle by expressions of the form

$$c = a - f \sin \theta \text{ for H corners}$$
$$d = b - g \sin \theta \text{ for E corners} \quad .. \quad (8)$$

Thus if the optimum widths are determined experimentally for the two right-angle bends, the constants f and g are known, and these expressions may then be used to interpolate for smaller angles of bend. The tolerance on the width is naturally greater for smaller angles. Typical bandwidth curves of right-angle bends are shown in Fig. 7: the bandwidth curves of bends of smaller angle are even better. For use on wavelengths of about 10 cm, reflectionless right-angle corners in both planes have been made by die-casting. Their bandwidth has been sufficient to permit a standardized compromise value of the corner-width to be adopted acceptable for the radar equipments of each of the three fighting services, operating on quite widely separated frequency bands.

For 3-cm wavelengths, the close tolerance required on the width of the E-plane corner



is a serious disadvantage. For that reason the earlier suggestion of two half-bends was revived, and since the dimensions of the guide are in any case small, the increase in mechanical size is negligible. Bandwidth curves are shown in Fig. 8. The separation of the two half-bends is here so small that considerable coupling occurs between them. This mutual impedance effect doubtless contributes to the good bandwidth characteristic. Where even larger bandwidths are required the half bends may each be of the chopped-off corner type, separately matched at the mid-band frequency, and spaced to extend the bandwidth.

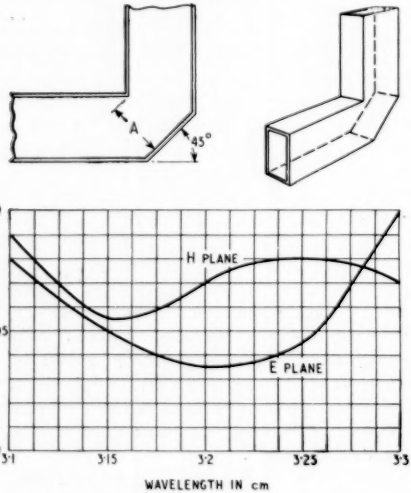


Fig. 7. *Bandwidths of right-angle corners. The optimum value of dimension A is 1.1 cm for E-plane corners and 2.5 cm for H-plane corners.*

8. T- and Y-Junctions

Waveguide T- and Y-junctions have been used for providing a continuous division of power into two arms of a waveguide system. More frequently in applications the power has been diverted into one or other of the arms alternately. This latter case constitutes a special type of bend in which the arm which is switched off has effectively a short-circuit across it, and thus acts as a reactive side stub at the corner. Junctions of this type have been matched by the arrangement of Fig. 9(a) which clearly shows the family resemblance to the chopped-off corner. The two dimensions d_1 and d_2 were determined experimentally: a typical H-plane junction of this type had a standing-wave ratio no

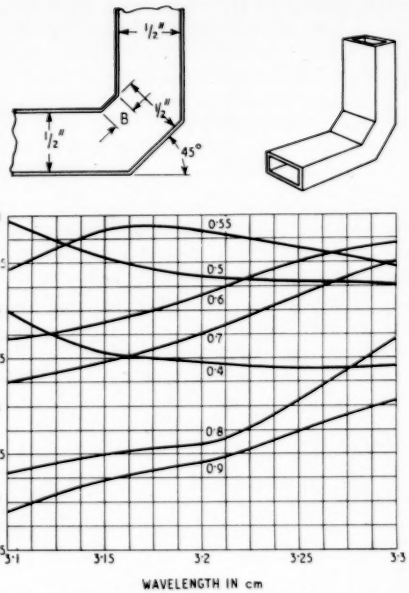


Fig. 8. *Bandwidths of right-angle corner composed of two half-bends. The figures on the curves show the dimension B in cm,—the optimum being 0.55 cm.*

worse than 0.90 over a $\pm 2\%$ frequency band. In a similar manner a Y-junction required for a special application was constructed as in Fig. 9(b), the dimension V being taken as a variable to be determined experimentally.

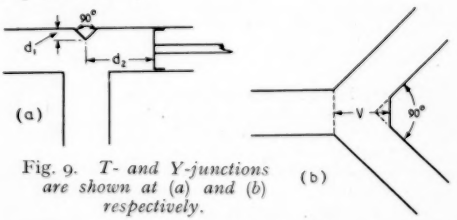


Fig. 9. *T- and Y-junctions are shown at (a) and (b) respectively.*

The simple unmodified T-junction of Fig. 10(a) has been analysed theoretically by Frank and Chu[4] for a junction in the E-plane, where the side arm is attached to the wider surface of the guide. The author has shown that these results may be expressed by the equivalent six-terminal network of Fig. 10(b). The impedances connected to or appearing at each pair of terminals are to be taken as the impedances at points in the corresponding waveguide arms sufficiently far from the junction to be clear of evanescent

modes but referred back by the usual transmission line impedance transformation to the plane ON for the collinear arms, and to the plane LM for the side arm. For convenience, the equivalent circuit parameters are expressed as specific admittances, being the admittance relative to the characteristic admittance of the transmission line. Their magnitudes depend on the ratio of the narrow dimension of the guide to the guide wavelength, and are shown plotted in Fig. 11.

Many alternative circuits can be derived which will have identical impedance transforming relations. One of these is shown in Fig. 10(c) and is very appropriate for considering the case when the side arm is closed by a short-circuiting piston and used as a side stub in series with the line.

Some special cases are of interest. First, if the side tube is closed at such a point that it introduces a specific susceptance $+j(f_2-f_1)/f_3$ at LM, then the junction introduces no reflection for the straight-through guide, but its electrical length is shortened by $2\tan^{-1}(1/f_2)$. Secondly, if the side stub introduces at LM, a susceptance $-j(1+f_1f_2)/f_2f_3$ then the admittance at ON presented to one limb of the straight through section is $1/jf_2$ whatever the value of the admittance on the remaining limb. Lastly, if one limb of the straight-through section is a stub presenting a susceptance $-1/jf_2$ at ON, then the admittance at ON presented to the other limb is $1/jf_2$ whatever the value of the admittance on the side tube. Similarly the admittance at LM presented to the side

Fig. 10 (below.) *The simple T-junction (a) may be represented by the equivalent six-terminal network (b). An alternative equivalent circuit is (c). The transformers are shown with typical turns ratios ($f_3 \geqslant 2$). The electrical lengths (θ) of the lines in (c) are given by* $\tan\theta = -1/f_2$.

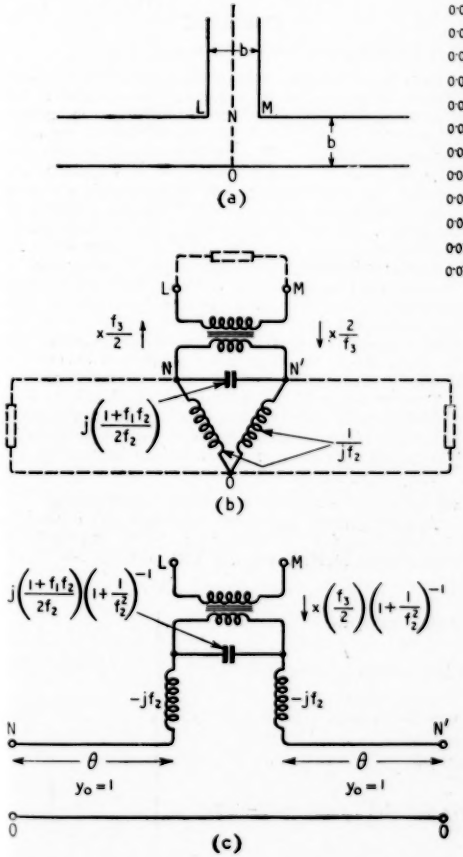
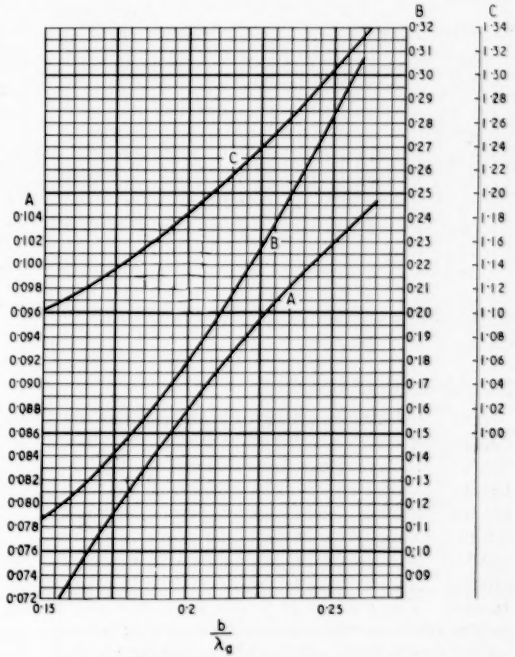


Fig. 11 (right). *The magnitude of the elements of the equivalent circuits of Fig. 10 are shown here. Scale A gives the susceptance of inductance ON* $(1/f_2)$, *while B indicates the susceptance of capacitor NN'* $\left(\frac{1+f_1f_2}{2f_2}\right)$; *scale C is for the transformer ratio* $(f_3/2)$.



tube is $+j(1+f_1f_2)/f_2f_3$ whatever the value of the admittance on the other straight-through limb.

It should also be noted that with one limb of the straight-through section forming a stub it is impossible to get transmission round the corner without reflection at the junction.

9. Conclusion

The foregoing review of waveguide components covers but a few of the elements used in waveguide systems for use in radar equipment: even these have by no means been exhaustively reviewed, but attention has been confined mainly to aspects in which the author has been in some way concerned. Among the specialized waveguide elements omitted may be mentioned rotating joints, attenuators, resistive loads and power measuring or monitoring devices, and couplings to other special devices and to waveguide radiators.

10. Acknowledgments

During the war, waveguide techniques and components for use in radar equipment were the subject of research and development by teams of workers at each of the three radar establishments. The early research work, when waveguides were still generally regarded as curiosities, was effected with the closest collaboration between these three teams. The author is aware how much this paper owes to the sum of waveguide knowledge which this collaboration produced, and wishes to acknowledge his indebtedness to these fellow workers in the teams led by Mr. E. Wild at the Admiralty Signals Establishment (A.S.E.), by Dr. W. D. Allen at the Telecommunications Research Establishment (T.R.E.), and by Dr. J. Ashmead at the Radar Research and Development Establishment (R.R.D.E.). The author would especially express his thanks to Dr. Ashmead, under whose guidance and encouragement he worked at R.R.D.E., and to Mr. L. A. Kerridge who assisted with the experimental work.

In the illustrative results quoted in the paper, use has been made of information from the following sources:—

Figs. 7 and 8 from a T.R.E. report by W. D. Allen and J. E. Williams, October 1942.

Fig. 9(a) from an A.S.E. memorandum by E. Wild, July, 1942.

Fig. 9(b) from a T.R.E. report by W. D. Allen, February, 1943.

REFERENCES

1 Pryce, M. H. L.: "Waveguides." *J. Instn. elect. Engrs.*, Vol. 93, Part IIIA, No. 1, p. 33 (1946).

2 See, for instance, the convenient summary in:—Lamont, H. R. L.: "Wave Guides" (Methuen & Co., Ltd.).

3 Schelkunoff, S. A.: *Bell System Technical Journal*, Vol 17, p. 17 (1938).

4 Frank, N. H. and Chu, L. J.: Massachusetts Institute of Technology reports, 1943-6, 7.

APPENDIX

13.1 *Electric plane bend or E-bend*

For the analysis, take cylindrical polar co-ordinates as indicated in Fig. 12. Let E and H be the electric and magnetic field vectors, a subscript denoting the component parallel to the corresponding co-ordinate axis. The mode corresponding to the $H_{1,0}$ mode in a straight guide may be qualitatively derived by considering the latter's characteristic field distribution to be distorted by the bending of the guide. It is evident that E_z will be zero while the transverse E may give rise to components E_r and E_θ: similarly the transverse H will yield a component H_z, while the axial H may yield components H_r and H_θ. Also it is to be expected that the sinusoidal distribution of the amplitude of the field across the width of the guide (broad side) will be maintained, whereas the distribution across the height (narrow side) will be no longer uniform.

The waves will be plane waves, having their wavefronts in the radial planes, and propagating with a uniform angular velocity round the curve. The wave expression may therefore be written:—

$$\exp.\ (j\omega t - jp\theta) \quad .. \quad .. \quad .. \quad .. \quad (9)$$

where p is the angular wavelength constant.

Applying these conditions to Maxwell's equation, the solutions for the field components, expressed in rationalized M.K.S. units, are found to be:—

$$\left.\begin{aligned}
E_r &= \left(\frac{1}{\beta r}\right)\left[A.J_p(\beta r) + B.N_p(\beta r)\right] .\ \sin(kz)\ .\ \exp.\ (j\omega t - jp\theta)\\
E_\theta &= \left(\frac{1}{p}\right)\left[A.J_p'(\beta r) + B.N_p'(\beta r)\right] .\ \sin(kz)\ .\ \exp.\left(j\omega t - jp\theta - j\frac{\pi}{2}\right)\\
H_\theta &= \left(\frac{k}{\omega\mu}\right)\left(\frac{1}{\beta r}\right)\left[A.J_p(\beta r) + B.N_p(\beta r)\right] \cos(kz)\ .\ \exp.\left(j\omega t - jp\theta + j\frac{\pi}{2}\right)\\
H_r &= \frac{-k}{\omega\mu p}\left[A.J_p'(\beta r) + B.N_p'(\beta r)\right] \cos(kz)\ .\ \exp.\ (j\omega t - jp\theta)\\
H_z &= \left(\frac{\beta}{\beta_0}\right)\left(\frac{1}{p}\right)\sqrt{\frac{\epsilon}{\mu}}\left[A.J_p(\beta r) + B.N_p(\beta r)\right] \sin(kz)\ .\ \exp.\ (j\omega t - jp\theta)
\end{aligned}\right\} \quad .. \quad (10)$$

where $J_p(\beta r)$ and $N_p(\beta r)$ are the Bessel functions of the first and second kinds, and the primes denote differentiation with respect to their argument (βr).

$k = \pi/a$ for the $H_{1,0}$ mode.

$\beta = \sqrt{\beta_0^2 - k^2} = 2\pi/\lambda_g$ = wavelength constant for a straight guide of the same cross-section.

$\beta_0 = \omega\sqrt{\mu\epsilon} = 2\pi/\lambda$ = wavelength constant for waves in free space.

μ and ϵ are the permeability and permittivity for the medium inside the tube, and have, respectively, the values $4\pi \times 10^{-7}$ henrys per metre and $10^{-9}/36\pi$ farads per metre for free space.

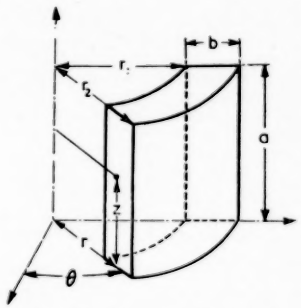


Fig. 12 (above). *Co-ordinate system for curved bends. (The dimensions a and b are shown for an E-bend; for an H-bend they should be interchanged).*

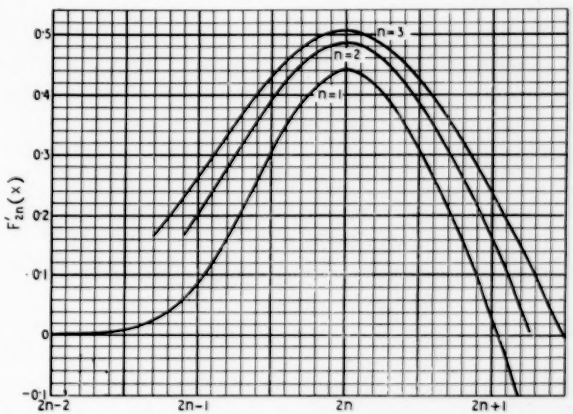


Fig. 13 (right). *Design graph for curved E-bend.*

The ratio of the constants A and B must be chosen to satisfy the boundary condition that E_θ is zero at $r = r_1$ and $r = r_2$: thus

$$-B/A = \frac{J_p'(\beta r_1)}{N_p'(\beta r_1)} = \frac{J_p'(\beta r_2)}{N_p'(\beta r_2)} \quad .. \quad .. \quad .. \quad (11)$$

Hence, if the dimensions of the curved guide are given, this equation determines the angular wavelength constant p. It may be noted that the plane of zero transverse current, where E_θ is zero, is the central plane $z = b/2$.

As the mean radius becomes large E_θ and H_r become small, while the three remaining components E_r, H_θ, H_z become substantially uniform in the direction of r, thus tending to the fields for the $H_{1,0}$ mode in a straight guide.

In the design of a bend which is to produce little reflection p is fixed and equation (11) then determines the mean radius for a given height of guide $b = (r_2 - r_1)$. A reflectionless bend must be of electrical length $n\pi$, where n is an integer, and for a right-angle bend it follows that

$$n\pi = p\theta = p\pi/2.$$

Hence

$$p = 2n; \quad n = 1, 2, 3 \ldots \quad .. \quad (12)$$

The functions $F'_{2n}(x) = \dfrac{J'_{2n}(x)}{N'_{2n}(x)}$ are shown plotted in Fig. 13.

Equation (11) requires that $F_{2n}(\beta r_1) = F_{2n}(\beta r_1 + \beta b)$. Since for a given cross-section β is known, this equation determines the inside radius (r_1) of the bend. For most practical cases the range $r_1 < r < r_2$ is of sufficiently small size about the maximum at $x = 2n$ for this part of the graph to be regarded as symmetrical, with the result that:—

$$\beta r_1 \approx 2n - \frac{\beta b}{2}$$

$$\beta r_2 \quad 2n + \frac{\beta b}{2}$$

and hence

$$\pi/4\,(r_1 + r_2) \approx \frac{n\pi}{\beta} = n\,\frac{\lambda_g}{2}.$$

This means that as an approximate solution, the mean length of the guide round the curve may be made equal to an integral number of the straight guide half-wavelengths. The accurate solution should be found by solving equation (11) with the aid of the graphs of Fig. 13. It should be noted that the mean guide wavelength for the curve may be less than, equal to, or greater than the guide wavelength for the corresponding straight guide, depending on their common cross-section.

13.2 *Magnetic Plane Bend or H-bend.*

By applying similar qualitative arguments to those of the last section, it is to be expected that the $H_{1,0}$ mode in the H-bend curved guide will have only the field components E_z, H_θ, H_r; that the fields will be uniform in the z direction; and that the sinusoidal distribution across the width of the guide will undergo modification.

The field vectors turn out to be:—

$$E_z = \left[A.J_p(\beta_0 r) + B.N_p(\beta_0 r)\right] \exp.\,(j\omega t - jp\theta).$$

$$H_r = \sqrt{\frac{\epsilon}{\mu}}\left(\frac{p}{\beta_0 r}\right)\left[A.J_p(\beta_0 r) + B.N_p(\beta_0 r)\right] exp.\,(j\omega t - jp\theta).$$

$$H_\theta = \sqrt{\frac{\epsilon}{\mu}}\left[A.J_p'(\beta_0 r) + B.N_p'(\beta_0 r)\right] exp.\,(j\omega t - jp\theta - j\pi/2)$$

The boundary condition $E_z = 0$ at $r = r_1$ and $r = r_2$ determines the ratio of B to A, and the value

of p when the dimensions of the curved guide are known :—

$$-B/A = \frac{J_p(\beta_0 r_1)}{N_p(\beta_0 r_1)} = \frac{J_p(\beta_0 r_2)}{N_p(\beta_0 r_2)} \quad (13)$$

As before if a right-angle bend is to be of electrical length $n\pi$ to eliminate reflection, then p is equal to $2n$. The functional equation (13) has been plotted in Fig. 14 to show $\beta_0 \frac{(r_2 + r_1)}{2}$ as a function of $\beta_0 (r_2 - r_1)$.

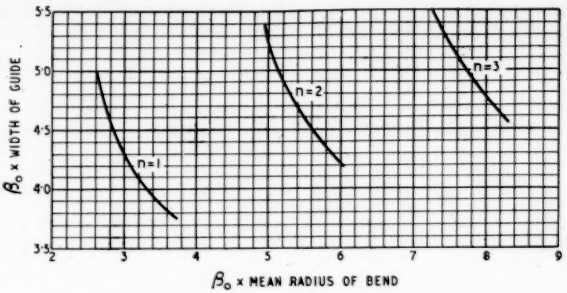


Fig. 14. *Design graph for curved H-bend.*

The position of the maximum value of transverse electric field (E_z), and of the zero transverse current is given by :—

$$-\frac{B}{A} = \frac{J_p'(\beta_0 r)}{N_p'(\beta_0 r)}$$

All the other quantities being known, this value of r, which is the correct plane in which to cut the guide if it is to be made in two parts, may readily be found for a limited but useful range of the variable, from the graphs of Fig. 13.

This circle of zero transverse current is always displaced outwards from the mean circle, and the wavelength along it is slightly longer than the wavelength in the straight guide.

DIVERSITY RECEPTION*

Statistical Evaluation of Possible Gain

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(*Signals Research and Development Establishment, Ministry of Supply, Christchurch*)

SUMMARY.—The diversity gain is defined as the reduction of transmitter power permissible on the introduction of a diversity system for the same proportion of time loss of signal.

Curves and formulae are obtained for this gain, which is shown to depend on the proportion of time loss allowed for the service, the number of received signals used, the correlation between fading in these signals and a parameter characterizing the propagation conditions. This parameter is discussed ; in order to obtain a better estimate of its value, it is suggested that more data on propagation conditions should be accumulated for links on which diversity reception is contemplated.

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2. Probability Integral of Signal Amplitude.
3. Evaluation of Diversity Gains.
   3.1. Gaussian Distribution of Signals.
   3.2. Rayleigh Distribution of Signals.
4. Power Law Approximation to the Probability Integral.
   4.1. The Evaluation of Gain.
   4.2. The Effect of Propagation Conditions on Gain.
5. Conclusions.

References.

Appendix.

1. Introduction

DIVERSITY reception makes use of the fact that signals do not, in general, fade simultaneously, whether received by spaced aerials, on diverse frequencies or by any other diversity system. Usually, at any instant, the strongest signal is utilized while the others are suppressed. Two constituent signals X, Y and the resulting diversity signal Z, will first be considered and, where possible, this consideration will be extended to the case of a greater number of signals.

One method of assessing the merit of a diversity system consists of examining the records of X, Y and Z, and finding the proportion of time during which the signals are below the receiver sensitivity. On referring to Fig. 1 (*a*), it will be seen that for the X-signal this entails the summation of the intervals i_1, i_2, . . . and dividing the sum by the length of the record. Similarly, for the Y-signal, the summation of the

* MS. accepted by the Editor, June, 1946.

intervals j_1, j_2, . . . is required. It will be noticed that with the diversity system the proportion of time loss could be found from the X- and Y-records by summing the intervals during which both signals are below the level of receiver sensitivity simultaneously, i.e., k_1, k_2, . . ., and dividing the sum by the length of the record. In this way an idea of the improvement in reception is obtained from the relative magnitudes of these proportions.

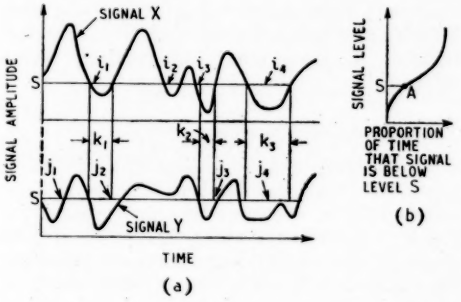


Fig. 1. *Typical curves of signal amplitude-time are shown at (a) for channels X and Y. The proportion of the time for which the X-signal is below the level S is indicated at (b).*

If the signal fluctuations are random, a probability distribution of signal strength may be considered and the probability integral curve constructed from the record. The probability integral represents the proportion of time that the signal lies below a certain level, and its value is found as explained above. The construction is illustrated in Fig. 1(*b*) where a typical form of the probability integral curve is shown; for example, the point A was obtained by summing the i's, [Fig. 1 (*a*)] and dividing by the length of the record. In some diversity systems (e.g., spaced aerials) it is reasonable to suppose that with a long enough record the X- and Y-probability integrals will not differ from each other appreciably; i.e.,

$$\Sigma i_n \approx \Sigma j_n$$

for all signal levels. These systems will be considered first. Fig. 2 gives typical curves of the probability integral as a function of the signal level s for the X-, Y- and Z-signals (the axes are here interchanged).

In practice the improvement in time loss is not the most important feature in diversity systems. If a given proportion of time loss can be tolerated in a communication service, the diversity system will allow a reduction in transmitted power, or possibly a relaxation in the requirements for receiver sensitivity. Referring to Fig. 2, it is seen that with a single receiver accepting a minimum signal level s_1, the time loss of signal would equal that for a diversity system using two receivers with a minimum signal level s_2. This means that the diversity system would allow the use of receivers of lower sensitivity. On the other hand, if the receiver sensitivity is kept constant, a reduction of transmitter power by a factor $(s_2/s_1)^2$ would be allowed. This reduction represents a merit of the diversity system and will be called "diversity gain," or simply "gain." It will be convenient to express it in decibels as

$$G = 20 \log_{10}\left[\frac{s_2}{s_1}\right] \text{db} \quad .. \quad .. \quad (1)$$

It is interesting to note that a simple extension of the procedure for finding the proportion of time loss for a given receiver sensitivity has led to the use of a probability integral. This suggests that some further conclusions may be obtained from a statistical consideration. Formulae for the diversity gain will be derived in this way.

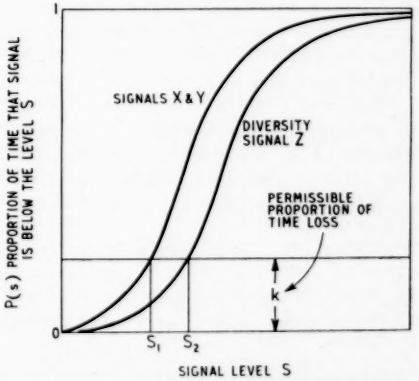


Fig. 2. *Typical probability-integral curves for the X- and Y-channels and the combined channel Z.*

2. Probability Integral of Signal Amplitude

Let the amplitudes of the signals X, Y and Z be specified at any time by the positive variates x, y and z, respectively. Denote the probability that the X- and Y-signals have amplitudes lying between x and $x + dx$, y and $y + dy$ simultaneously, by $\phi(x, y)\,dxdy$. The probability that the X-signal lies between x and $x + dx$ is denoted by

$$p_x(x) \cdot dx = dx \int_0^\infty \phi(x, y) \cdot dy \quad .. \quad (2)$$

and the probability integral of the X-signal is denoted by

$$P_x(s) = \int_0^s p_x(x) \cdot dx \; . \quad .. \quad .. \quad (3)$$

Similar expressions apply to the Y-signal.

If the stronger signal suppresses the weaker, as is usually the case, the signal Z will always be less than s, say, whenever x and y are both less than s. Consequently, the probability integral of the Z-signal is

$$P_z(s) = \int_0^s \int_0^s \phi(x,y) \cdot dxdy \; . \quad .. \quad (4)$$

When the signals fade independently, $\phi(x, y)$ is separable and is equal to $p_x(x) \cdot p_y(y)$, and so

$$P_z(s) = \left\{ \int_0^s p_x(x) \cdot dx \right\} \left\{ \int_0^s p_y(y) \cdot dy \right\}$$
$$= P_x(s) \cdot P_y(s), \quad .. \quad (5)$$

the product of individual probability integrals. In particular, if $P_x(s) = P_y(s) = P(s)$, say,

$$P_z(s) = [P(s)]^2, \quad .. \quad .. \quad .. \quad (5a)$$

the square of non-diversity probability integrals. For an n-diversity system in which there are n independent signals, $X_1, X_2, \ldots X_n$, this last result becomes $P_n(s) = [P(s)]^n$.

Summing up, at any signal level, the proportion of time loss in a diversity system with independent signals is the product of the proportions of time loss for the constituent signals.

3. The Evaluation of Diversity Gain

In order to evaluate the diversity gain a probability distribution must be assumed. The most typical distributions are the Gaussian, or normal, law and the Rayleigh law.

When the fading is shallow, the distribution is often approximately normal; this would also be true if the fading were produced by the combined action of a large number of small independent perturbations of the r.f. signal. If x is the deviation of the signal amplitude from its mean value, the law has the form

$$p_x(x)\, dx = \frac{1}{\sigma(2\pi)^{\frac{1}{2}}} \exp\left\{\frac{-x^2}{2\sigma^2}\right\} \cdot dx, \quad (6)$$

where σ is the r.m.s. deviation of the signal from the mean.

When the fades are deep, the probability distribution often approaches the Rayleigh distribution

$$p_x(x)\, dx = \frac{2x}{\sigma^2} \exp\left\{\frac{-x^2}{\sigma^2}\right\} \cdot dx, \quad x \geqslant 0, \quad (7)$$

where σ is the r.m.s. value of the signal measured from zero. This law would also hold if the r.f. signal were the sum of a great number of components in random phase relation.[1]

3.1. *Gaussian distribution*

A disadvantage of using the Gaussian probability function is that it extends from $-\infty$ to $+\infty$, whereas the signal, being the result of detection, cannot change sign. The Gaussian law could never, therefore, represent the facts exactly, but will at least yield an approximation. On the other hand, it has the advantage that the effect of correlation between the constituent signals can be estimated comparatively simply. When the receiving aerials in a space-diversity system are closely spaced, the signals will generally fade simultaneously. By increasing the spacing, the correlation between the signals may be reduced. In some conditions it may even be possible to have negative correlation; in that case a decrease of one signal is often accompanied by an increase of the other. In general, however, one may expect at best to reach zero correlation. It will be seen later how a decrease of the correlation increases the diversity gain. Other known ways of obtaining signals with low correlation are frequency and polarization diversity, and the Musa system in which signals are isolated according to the number of ionospheric reflections suffered.

The bivariate Gaussian distribution for two variables x and y, may be written in a normalized form, where the mean values of x and y are zero and the r.m.s. values are unity:

$$\phi_g(x, y) = \frac{1}{2\pi(1-\rho^2)^{\frac{1}{2}}} \exp - \left\{\frac{(x^2 - 2\rho xy + y^2)}{2(1-\rho^2)}\right\} \quad (6a)$$

where ρ is the correlation coefficient; its value has a marked effect on the gain that may be expected from a diversity system.

To determine this influence, it is necessary to find the probability integral

$$P_z(s) = \int_{-\infty}^{s}\int_{-\infty}^{s} \omega_\rho(x, y) \, . \, dxdy, \quad .. \quad (4a)$$

which will be a function of ρ; here x, y and s are measured from the mean signal level.

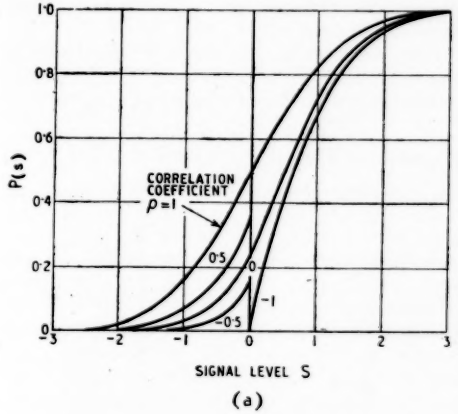


Fig. 3. *Normal-law probability-integral curves are shown at (a) and the lower part to an enlarged scale at (b).*

It can be shown[2] that the integral in (4*a*) is

$$P_z(s) = \frac{1}{4}\left[1 + \text{erf}\, s/\sqrt{2}\right]^2 + \frac{1}{2\pi} e^{-s^2} \sum_{r=1}^{\infty} \left[H_{r-1}(s)\right]^2 \frac{\rho^r}{r!} \quad .. \quad (4b)$$

where

$$\text{erf}\, \alpha = \frac{2}{\sqrt{\pi}} \int_0^{\alpha} e^{-t^2} dt$$

and $H_r(s)$ is the r-th Hermite polynomial. Computed results are shown in Fig. 3(*a*) for the following values of ρ:

(*a*) $\rho = 1$. Here the X- and Y-signals have identical amplitudes and no gain is obtained. The probability integral $P(s)$ reduces to the simple form $\frac{1}{2}(1 + \text{erf}\, \frac{1}{2}s)$.

(*b*) $\rho = 0$. This corresponds to independent signals and $P(s)$ reduces to $[\frac{1}{2}(1 + \text{erf}\frac{1}{2}s)]^2$.

(*c*) $\rho = -1$. In this case no losses of signal would be expected below the common mean, since one signal is the mirror image of the other. Consequently the probability integral is zero below the mean signal level.

(*d*) The intermediate cases, $\rho = 0.5$ and -0.5 are shown for negative s only.

The lower part of the curves is also shown in Fig. 3 (*b*), in which the vertical axis is marked in percentages on a logarithmic scale.

The improvement may be found directly from these curves, but the gain cannot be determined unless the mean signal value is known; this corresponds to the zero co-ordinate in Fig. 3 (*a*) and (*b*). In this case, the true zero of the horizontal scale can be inserted and the gain obtained from (1). Two examples have been computed and the results are shown in Fig. 4. The curves are given as functions of ρ for the permissible proportion of time loss $k = 0.5$, 1, 2 and 5 per cent, for the two cases when the ratio of the mean to the root-mean-square variation about the mean is 3 and 5.

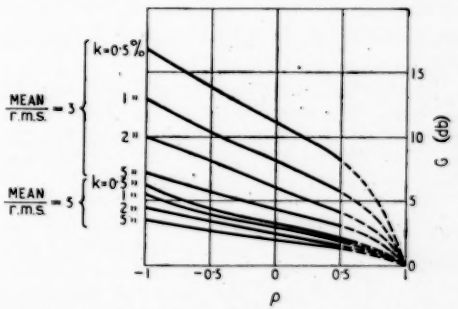


Fig. 4. *Examples of diversity gain G(ρ).*

3.2. *Rayleigh distribution*

Unlike the Gaussian law, the Rayleigh probability law permits only positive values of the signal. The distribution is completely described by one parameter. Its mean value is given by $x = \frac{1}{2}\sigma\sqrt{\pi}$ where σ is the root-mean-square value of the amplitude measured from zero. In the bivariate case,

this law is less tractable analytically and only the case of independent signals will be considered here.

The probability integral is easily obtained; assuming that $\sigma = 1$,

$$P(s) = \int_0^s 2xe^{-x^2} \cdot dx = \left[1 - e^{-s^2}\right]. \quad (8)$$

For a 2-diversity system

$$P(s) = \left[1 - e^{-s^2}\right]^2, \quad .. \quad .. \quad (8a)$$

and in general with an n-diversity system

$$P(s) = \left[1 - e^{-s^2}\right]^n. \quad .. \quad .. \quad (8b)$$

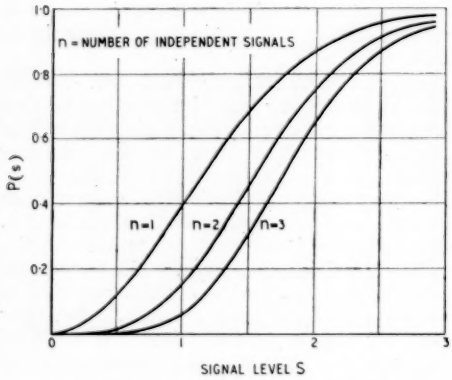


Fig. 5. *Rayleigh-law probability-integral curves.*

These functions are shown in Fig. 5 for the cases $n = 1$, 2 and 3.

The formula for diversity gain will next be derived. Suppose that for a permissible proportion of time loss k, we have signal levels s_1 on the non-diversity and s_n on the n-diversity system. Then

$$k = 1 - e^{-s_1^2} = \left[1 - e^{-s_n^2}\right]^n, \quad .. \quad (9)$$

and so

$$\frac{s_n}{s_1} = \left\{\frac{\log(1 - k^{1/n})}{\log(1 - k)}\right\}^{\frac{1}{2}}. \quad .. \quad .. \quad (10)$$

Hence the gain in decibels is

$$G = 20 \log_{10} s_n/s_1 = 10 \log_{10} \left\{\frac{\log(1-k^{1/n})}{\log(1-k)}\right\}. \quad .. \quad .. \quad (11)$$

Fig. 6 shows a graph of the gain, $G(k)$, for the cases $n = 2$ and 3.

The effect of increasing n is shown by the full lines in Fig. 7. Inspection shows that as the number of independent signals is increased, the gain is also increased but by a diminishing amount.

4. Power-Law Approximation to the Probability Integral

It has been implied in the previous section that the lower parts of the probability integral curves are of special interest since the permissible proportions of time loss are usually low.

Suppose that in this range of signal values the probability integral can be approximately represented by the power law

$$P(s) = \lambda s^m, \quad P(s) \ll 1,$$

where λ and m are constants. This leads

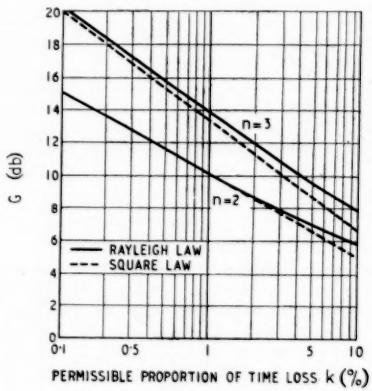


Fig. 6. *Diversity gain G (k) for two- and three-signal systems.*

to a simple formula for the diversity gain, easily applicable to practical cases.

4.1. *The evaluation of gain.*

The equation corresponding to (9) above is in this case

$$k = \lambda s_1^m = \left[\lambda s_n^m\right]^n, \quad .. \quad .. \quad (13)$$

where s_n, s_1 are the levels of the diversity and non-diversity signals respectively, corresponding to the given time loss k. Hence

$$G = 20 \log s_n/s_1 = \frac{20}{m}\left\{1 - \frac{1}{n}\right\} \log_{10} \frac{1}{k} \text{ db}. \quad (14)$$

For example, if the power-law approximation is valid for small k, and if a 1 per cent time loss of signal is allowed, the gain for independent signals will be $\frac{20}{m}$ db in a 2-diversity system. As the number of constituent

signals is increased, the gain at this level will rise asymptotically to $\frac{40}{m}$ db.

The range of validity of the power-law approximation must extend to a signal value which depends on the number of signals used, the diversity probability-integral curve and the value of k. Higher values of k and a greater number of signals require a greater range of validity. This can be seen from Fig. 8, where the Rayleigh probability integral curves (full lines) are drawn on a log-log scale on which a power law, as in (13), gives straight lines (broken lines). The range of linearity should extend at least to s_2, s_3 or s_2', s_3' if two or three signals and a level k or k' are considered. The effect of departure from linearity of the Rayleigh probability integral is illustrated in Fig. 6 and Fig. 7, where the gain is plotted for both the Rayleigh law and the square law approximation.

Up to this point the probability integrals, and therefore the mean values and standard deviations of the constituent signals, have been assumed identical at the point of the

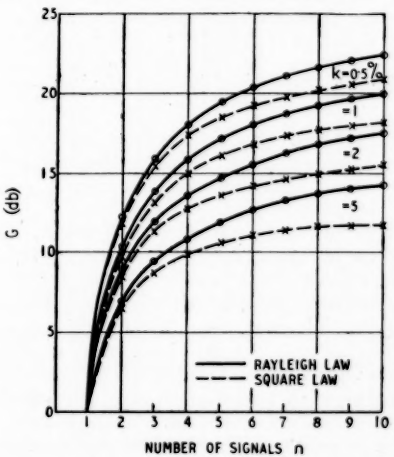


Fig. 7. *Effect on diversity gain of increasing the number of signals.*

circuit where they are compared and the strongest selected. This is not always true. For instance, the receivers may have different sensitivities or the aerials different gains. However, all signals may still retain the same value of m.

It is shown in the Appendix that in this case (14) gives the diversity gain of the system over a single receiver which delivers a signal having a strength equal to the geometric mean of the strengths of the actual constituent signals when expressed in voltages, or alternatively, their arithmetic mean if expressed in decibels. It follows, for example, that if the sensitivity of one of the receivers changes, it will affect the reception only in so far as it alters the mean sensitivity of all receivers.

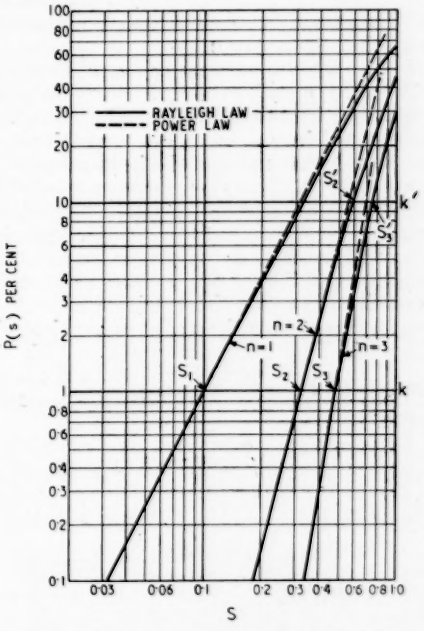


Fig. 8. *Range of linearity of Rayleigh law on log-log scale.*

It can be shown that the above conclusions only hold for a probability integral given by a power law.

A consideration of the case when the signals have neither the same mean value nor the same value of m did not yield a simple and convenient formula.

4.2. *The effect of propagation conditions on gain.*

It can be seen from (14) that the diversity gain depends on three parameters; two of them, n and k, have already been discussed; n can be determined by the designer, k is chosen by the user of the system. The remaining parameter m, however, depends on propagation conditions and is usually diffi-

cult to control. Moreover, its value varies with time.

In order to get an idea of the probable values of m, some theoretical considerations can be made, but it is more prudent to use empirical results.

If the received signal is considered as the result of the superposition of a number of signals of equal amplitudes and in random phase relation, the values of m can be derived from the formulæ and curves given by M. Slack.[8] In the case of two signals $m = 1$ and for more than two, $m = 2$.

For an infinite number of components, not necessarily equal in amplitude, the probability distribution follows the Rayleigh law in which case $m = 2$. A graph paper has been prepared in which the Rayleigh probability integral (8) appears as a straight line at 45° to the axes; the horizontal signal scale s is logarithmic.* Since the Rayleigh law is approximately linear for small s, the lower part of the vertical scale is very nearly logarithmic, so that the graph paper may be used to determine the value of m when the Rayleigh law does not apply. With the scales used the value of m is twice the slope of the lower part of the probability integral curve.

Experimental examples of probability integral curves are plotted on this graph paper in Fig. 9 for two records made at an N.P.L. station by J. H. Tait and N. Creed (S.R.D.E.).

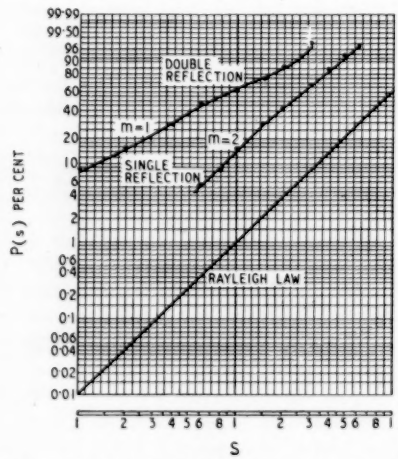


Fig. 9. *Probability integrals of single- and double-reflections of pulses from the F-layer plotted on Rayleigh probability paper.*

* For convenience a linear decibel scale is used.

Pulse transmission at 1,025 kc/s was received over a distance of 116 km during the night. Two of the pulse trains received were identified as being the result of one and two reflections from the F-layer of the ionosphere respectively. Single reflection gave a probability integral which approximated closely to the Rayleigh form, thus giving $m = 2$. Double reflection, on the other hand, was far from the Rayleigh law, giving $m = 1$ in the lower part. A straight line at 45° to the axes, corresponding to the Rayleigh law, is drawn for comparison.

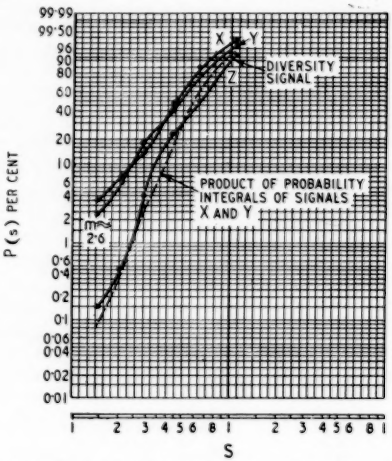


Fig. 10. *Probability integrals of signals on a 7-cm link.*

Other examples are given in Fig. 10 from records obtained by W. Childs at S.R.D.E. experimental stations in Wales, for transmission at a wavelength of about 7 cm on a 60-mile link over sea. Two receivers gave signals X and Y and these were combined to give a diversity signal Z. The probability integrals of X and Y plotted in Fig. 10 show some deviation from the Rayleigh law and have $m = 2.6$. The product of these probability integrals is plotted as a broken line; the diversity probability integral deviates little from this, indicating that the signals X and Y were substantially independent.

Similar records obtained in Wales at various times when heavy fading occurred gave several values of m. A few of them, chosen at random, are given below:—

1.9; 1.9; 1.2; 3.0; 1.4; 1.7; 5.9; 1.8; 2.5; 1.4; 1.2; 1.3; 2.2; 2.2; 2.5.

The mean value of these is about 2.1,

but there is a considerable scatter ; an even greater scatter was obtained from other records of pulses reflected from E- and F-layers in the experiments at N.P.L. mentioned earlier. It may be noticed, however, that the cases in which m is large, giving a small value of G, may be discounted since then the fading is usually shallow, so that reception is satisfactory without a diversity system.

It was mentioned in Section 3 that when the fades are shallow the distribution is usually approximately normal. This is illustrated in Fig. 11 where a probability integral is plotted from a record obtained over a 10-cm link by S. Jarkowski at the stations in Wales mentioned above. It is plotted on the so-called " Arithmetic Probability " paper in which the probability integral of a Gaussian distribution would appear as a straight line.

The limited number of examples shown above gives no more than an indication of the values of m likely to be encountered. From these it appears that $m = 2$ may serve as a working value until more information is

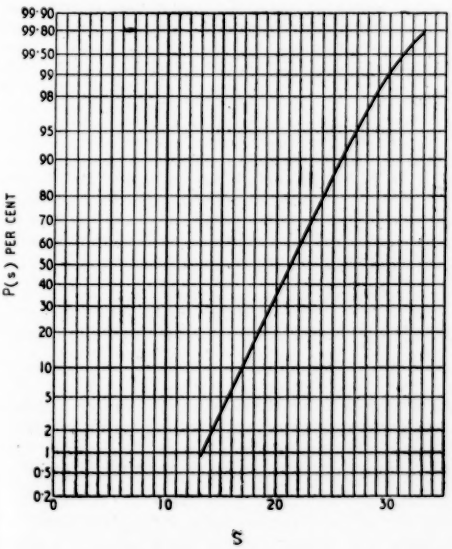


Fig. 11. *Probability integral of a 10-cm signal with shallow fading.*

obtained. More records should be taken on links over which diversity reception may be needed, the probability integrals should be constructed as described here, plotted on log-log or Rayleigh paper and values of m accumulated for a statistical study. In practice the mean signal level varies from time to time, and diversity reception is most needed when the signal is weakest. Consequently, the periods with strong signals should be ignored.

5. Conclusions

If the constituent signals of a diversity system fade independently, the proportion of time in which the diversity signal falls below receiver sensitivity is the product of the corresponding proportions for the constituent signals. In a diversity system comprising n receivers of equal sensitivity, the proportion of time loss of signal will be the n-th power of the proportion for a single receiver.

If the constituent signals have the same probability integral and fade independently, the formula for diversity gain is approximately

$$G = \frac{20}{m}\left\{1 - \frac{1}{n}\right\} \log_{10} \frac{1}{k} \text{ db}$$

where k is the proportion of time loss allowed for the system, n is the number of constituent signals, and m is the slope of the lower part of the probability-integral curve of a signal when plotted on a log-log scale. It follows that, when the permissible proportion of time loss is smaller, a greater diversity gain may be expected. Also, as more signals are added, the gain is increased but each additional signal gives a reduced increase.

If there is some positive correlation between the fading of the constituent signals, the diversity gain is less than that given by the above formula ; it is greater in the case of negative correlation.

The limited experimental information available shows that the slope m assumes various values according to the propagation conditions. It appears that $m = 2$ may serve as a working value until more information is obtained.

It is suggested that data should be accumulated giving the values of m encountered on links for which diversity reception is contemplated.

If the received signals have different mean values but still the same value of m, the above formula gives the diversity gain of the system over a single receiver delivering a signal which has a mean value equal to the arithmetic mean of the actual constituent signals when expressed in decibels.

The permission of the Chief Scientist,

Ministry of Supply to publish this paper is gratefully acknowledged.

The authors wish to thank Dr. J. S. McPetrie for advice on the method of presentation of this paper. They are also indebted to their colleagues at S.R.D.E., Messrs. J. H. Tait, N. Creed, W. Childs and S. Jarkowski, whose experimental data were utilized.

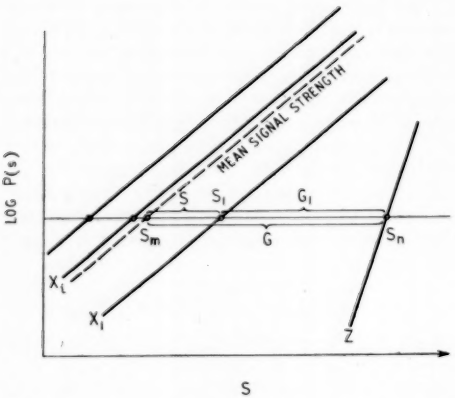


Fig. 12. *Probability integrals of signals of unequal strength.*

APPENDIX

The Effect of Unequal Strength of Signals on Diversity Gain

If the receivers of an n-diversity system deliver signals of different mean values, but with the same value of m, the corresponding power-law approximations to the probability integrals may be written

$$P_i(s) = \lambda (a_i s)^m$$

where $a_i \geqslant 1$ is the ratio of the mean values of the strongest signal and that from the i-th receiver. For the strongest signal $a_1 = 1$.

In Fig. 12 let X_1 be the probability integral of the strongest signal, plotted on log-log paper, and let the lines parallel to it correspond to the other signals. Let the line Z correspond to the diversity signal. The gain will be referred at first to the strongest signal.

If the signals fade independently, it follows, as in (13), that

$$\begin{aligned} k = \lambda s_1^m &= \lambda s_n^m \lambda (a_2 s_n)^m \ldots \lambda (a_n s_n)^m, \\ &= \lambda^n s_n^{mn} (a_2 a_3 \ldots a_n)^m, \end{aligned}$$

so that the gain in decibels is

$$\begin{aligned} G_1 &= 20 \log_{10} s_n/s_1, \\ &= \frac{20}{m}\left\{1 - \frac{1}{n}\right\} \log_{10} \frac{1}{k} \\ &\quad - \frac{1}{n}\left\{20 \log_{10} a_2 + \ldots + 20 \log_{10} a_n\right\}, \\ &= G - S, \end{aligned}$$

say, where G is the gain given by (14) and S is the difference between the mean value of the strongest signal and the mean strength of all signals, expressed in decibels. This is illustrated in Fig. 12.

It follows that (14) gives the gain of the diversity system over a single receiver delivering a signal with a strength which is the arithmetic mean of the strengths of the actual constituent signals if expressed in decibels, or alternatively their geometric mean if expressed in voltage.

REFERENCES

1 Lord Rayleigh, "On the Resultant of a Large Number of Vibrations of the Same Pitch and of Arbitrary Phase." *Phil. Mag.*, 1880, Vol. 10, p. 73.

2 Elderton, W. P.: "Frequency-Curves and Correlation." (C. & E. Layton) Appendix III.

3 "The Probability Distribution of Sinusoidal Oscillations Combined in Random Phase." *Journ. Instn elec. Engrs.*, 1946, Vol. 93, Part III, pp. 76–86.

CORRESPONDENCE

The Equivalent Diode

To the Editor, "Wireless Engineer"

SIR,—I was much interested in the above study by G. B. Walker, with which I mainly agree. Apart from an obvious typographical error resulting in a confusion of a and α in the final equation, my only criticism is the somewhat minor one that full justice has not been done to the formula obtained, in that it would appear ;

(a) that if $V_k = 0$, the μ used need not be the electrostatic μ but may be the true μ and thus depend on the electrode potentials. The restriction in range mentioned by Mr. Walker thus disappears ; which is just as well, as the range over which the true μ and the electrostatic μ coincide is really rather small.

(*b*) that if, indeed, μ is the electrostatic μ, and independent of electrode potentials, then we are not restricted to $V_k = 0$.

In order to establish the above, all we need do is to allow for the variation of μ with V_k, so that

$$A\left(1 + \mu + V_k \frac{\partial \mu}{\partial V_k}\right) = A'$$

giving, for V_k not zero,

$$V_d = V_k + \frac{V_a - V_k + \mu(V_g - V_k)}{1 + \mu + V_k \partial\mu/\partial V_k}$$

Then if $V_k = 0$ we have, as obtained by Mr. Walker

$$V_d = \frac{V_a + \mu V_g}{1 + \mu}$$

which is seen to be true whether μ is variable or not.

Alternatively, if $\partial\mu/\partial V_k = 0$, we have

$$\begin{aligned} V_d &= V_k + \frac{V_a - V_k(1 + \mu) + \mu V_g}{1 + \mu} \\ &= \frac{V_a + \mu V_g}{1 + \mu} \end{aligned}$$

—again, Mr. Walker's result, but now not restricted to $V_k = 0$.

Mr. Walker's formula differs from mine by the factor $\sqrt{1 + \frac{1}{\mu}}$. Although in practice this factor is usually very close to unity, its discovery is an important theoretical feature, and in view of the special properties of tubes having a fractional μ, immediate practical application is not lacking.

P.R.T. Laboratries Ltd., W. E. BENHAM.
High Wycombe, Bucks.

Is Rotation Absolute or Relative ?

To the Editor, "Wireless Engineer"

SIR,—Rotation about any axis may be considered as rotation about any other parallel axis together with rectilinear motion at right angles to, and in the common plane of, these axes. The rectilinear motion is not absolute, so we need only consider rotation about, say, the centre of gravity of the system under observation. To the engineer it would seem that this may be detected and measured without reference to the external world, by means of a hypothetical instrument consisting of a pair of equal masses, m, attached to the ends of a massless rod which can be rotated about a central transverse axis, at a speed, ω, measured relative to the observed system. If latter has no absolute rotation a dynamometer embodied in the rod will register a force of $rm\omega^2$, where r is half the separation of the masses ; but if the system has a component of absolute rotation about the axis of the instrument the force will be correspondingly reduced or increased. By measuring this force with the instrument set in three successive directions it should be possible to calculate the direction and magnitude of absolute rotation of the system.

P. M. C. LACEY

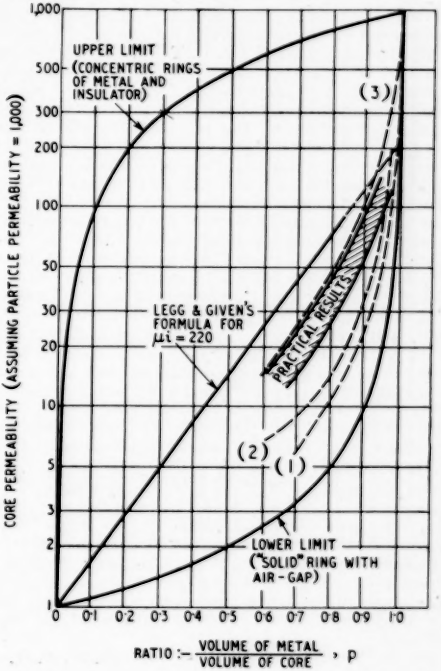
Aylesbury, Bucks.

Permeability of Dust Cores

To the Editor, "Wireless Engineer."

SIR,—The recent correspondence on the theoretical considerations of the permeability of dust cores has aroused considerable interest, but the practical aspect of the subject does not seem to have received the same attention.

Some years ago we were puzzled by the fact that the permeability of a dust core with a known percentage of filler was considerably higher than that calculated from the formula, assuming a simple air-gap or assuming the particles to be uniformly coated spheres. As Prof. Howe points out, the ultimate limit in permeability would be that of a core comprising laminations aligned in the direction of the flux. In practice we have found that over a limited range of percentage fillers the curve of core permeability against percentage filler agrees reasonably well with a formula calculated on the assumption that the particles are uniformly coated rectangular slabs (dimension ratio : $10 \times 2 \times 1$) of 1,000 permeability material aligned with the long axis of the slab in the direction of the field (see figure). Photomicrographs of core sections have shown that under the high pressures used the particles may become flattened and tend to align themselves (in a toroid) in the required direction. The greater the flattening the higher will be the permeability. A certain amount of prominence has been given in your correspondence to Legg and Given's[1] empirical formula $\log \mu = p \log \mu_i$. The value of μ_i (intrinsic permeability) assumed by them as being 220 (for molybdenum-permalloy powder) has been considered by us to be an arbitrary value inserted in the equation so that the equation shall represent as closely as possible the practical results over a limited range of core permeability (up to about 90) and should not be associated with actual permeability of the material. The values of intrinsic permeability are certainly much higher. The published values for the permeability of Mo-permalloy in sheet form are of the order of 22,000 as compared with 9,000 for Ni-Fe alloy.[2] We have made attempts to measure the permeability of a Ni-Fe alloy as used in dust cores and have obtained values of about 1,000 (measured on d.c.) for cores pressed at the usual working pressure with no insulation.



Relation between core-permeability and metal content for different arrangements of particles. Key to theoretical curves : (1) cubic particles ; (2) cubic particles (with corners cut off)—approximating to flattened spheres ; (3) rectangular slabs (dimension ratio 10 : 2 : 1), with the long axis parallel to the field.

There are still considerable air-gaps in such a core, as is evidenced by the fact that if cores are made from the material in such a way that air-gaps are largely avoided the permeability is about 3,000.

It would appear, therefore, that it is safe to assume a figure of, say, not less than 6,000 for the intrinsic permeability of molybdenum permalloy powder, and this coupled with a certain amount of particle flattening would seem to account largely for the so-called anomalously high core-permeabilities of upwards of 150 which may be encountered in practice.

These considerations, of course, represent only a small part of the rather complex problem of such a heterogeneous body as a dust-iron core.

G.E.C. Research Laboratories, Wembley. P. R. BARDELL.

[1] *Bell System Tech. Journal*, 1940, Vol. 19, p. 385.
[2] *Bell System Tech. Journal*, 1939, Vol. 18, p. 438.

NEW BOOKS

Der Frequenzstabile Schwingtopf-Generator

By ARNOLD BRAUN. Pp. 79, with 37 Figs. Published by Verlag A.G. Gebr. Leemann & Co. Zürich. Price 7.50 Francs (Swiss).

This is the author's thesis for the D.Sc. degree at Zürich Technische Hochschule. The work was carried out in the Institut für Hochfrequenztechnik under the direction of Dr. F. Tank. It deals with the use of a cavity resonator as the oscillatory circuit of a valve oscillator at frequencies of about 200 Mc/s, with special reference to the constancy of frequency and to the optimum design of the cavity resonator. The effects of temperature changes and mechanical stresses on the dimensions of the resonator and the effect of this and of air pressure and moisture on the frequency are discussed. The calculation of the losses and of the equivalent resistance and their effect on the dimensions necessary to give the optimum Q value are discussed very fully, as is also the effect of the connections between the resonator and the valve. The construction and testing of two cavity resonators, both connected to push-pull oscillators, in one case with inductive coupling and in the case with capacitive coupling, are described and discussed in detail. Figs. 4*a* and 4*b* do not agree with the text, and there seems to be something wrong with the definitions of permeability and dielectric constant on page 4. μ_0 is given as 1.256. $10^{-8}\left[\frac{A \text{ sec}}{V \text{ cm}}\right]$ and ϵ_0 as 0.0886. $10^{-12}\left[\frac{V \text{ sec}}{A \text{ cm}}\right]$. The quantities in brackets should be interchanged; this can be seen at once since $\mu_0 = 1.256 \cdot 10^{-8}$ henry/cm and $\epsilon_0 = 0.0886 \cdot 10^{-12}$ farad/cm. Instead of henrys one can put ϕ/A or V sec/A^2; similarly, instead of farads one can put Q/V or A sec/V. These are minor details, however, and the book can be recommended to anyone interested in this subject.

G. W. O. H.

"Wave Propagation in Periodic Structures"

By L. BRILLOUIN. Pp. 247 + viii, with 137 diagrams. Published by McGraw-Hill Book Co., Ltd., Aldwych House, London, W.C.2. Price 20s.

This is an unusual book, covering as it does problems in diverse and normally unrelated fields. The author's theme is the common mathematical background linking different types of wave motion —elastic waves, electromagnetic waves, and electron waves in wave mechanics—and in particular the similarities of their interaction with periodic structures like crystal lattices and electric-wave filters; the engineer and the physicist can learn from each other in their methods of approach to these problems.

The book starts simply with the consideration of oscillations in a chain of particles and their relations to line networks. This is then extended into a general theory of one-, two- and three-dimensional lattices of particles, and proceeds to a study of the zone concept in the theory of solids, to which the author has made important contributions. This, occupying about two-thirds of the book, is largely the domain of the physicist, and the radio engineer will sometimes be hard put to it to see similarities with the properties of filter networks. In the latter third of the book is presented the engineer's approach to networks through the matrix calculus. This is finally extended to the case of continuous electric lines, which are shown to involve matrices similar to those occurring in the Pauli theory of electron spin.

The book is fairly easy to read, but the similarities of method are often masked by the differences of approach. It is a book to browse through, not to study, and as such probably one should not complain that the treatment is sometimes scrappy and the progress somewhat random. The orthodox radio engineer may make little of it, but to those whose minds are more flexible it will provide valuable new outlooks and parallel avenues of thought in other fields. In this respect a criticism might be made of the scanty references to original sources. Often a man's name is quoted but not always a reference given to the work in question.

More attention is given to the atomic than to the macroscopic lattice, and the book would have benefited by a discussion, for example, of the radiation pattern of a three-dimensional array of dipoles, and its similarities with the X-ray diffraction pattern of a crystal. It seems unfortunate that the author does not mention the recent work of Kron and his collaborators in devising two- and three-dimensional networks as equivalent circuits for electromagnetic fields in bounded space. This would form a very fitting close to the book.

The volume is one of the well-known "International Series in Physics" but is smaller than normal in page size and contains only 247 pages. The typography is up to the usual high standards of McGraw Hill, but it gave the reviewer some slight comfort to find a 1946 U.S. publication which, in addition to the usual "economy of essential materials" notice on the half-title page, did show some signs of austerity in paper quality and margins.

H. R. L. L.

Broadcasting Stations of the World

Pp. 48. Iliffe & Sons Ltd., Dorset House, Stamford Street, London, S.E.1. Price 1s.

Compiled by *Wireless World* this booklet contains lists of broadcasting stations arranged both in order of frequency and by geographical location. European medium- and long-wave stations are included, and the short-wave lists cover the whole world and give call-signs as well as the power of stations.

THE PHYSICAL SOCIETY

A meeting of those interested in the formation of an acoustical group will be held in the Jarvis Hall of the Royal Institute of British Architects, 66, Portland Place, London, W.1, on 19th February, 1947, at 3 p.m. There will be a paper by Dr. Alex Wood on "The Contribution of Acoustical Science to Allied Studies," and a general discussion of the proposal to form an acoustical group.

Those interested are asked to send their names to the acting secretaries, A. T. Pickles and W. A. Allen, at 1, Lowther Gardens, Prince Consort Road, London, S.W.7.